## DEVELOPMENT OF A FUZZY TIME SERIES MODEL USING CAT SWARM OPTIMIZATION CLUSTERING AND OPTIMIZED WEIGHTS OF FUZZY

**RELATIONS**

## By

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## DECLARATION

I **ALHASSAN** Balarabe Mohammed, hereby declare that the work in this Dissertation entitled **“Development of a Fuzzy Time Series Model Using Cat Swarm Optimization Clustering and Particle Swarm Optimization”** has been carried out by me in the Department of Computer Engineering. The information derived from literature has been duly acknowledged in the text and a list of references provided. No part of this dissertation was previously presented for another degree or diploma at this or any other institution.

............................................................ ....................................

## ALHASSAN Balarabe Mohammed Date

**CERTIFICATION**

This Dissertation titled “DEVELOPMENT OF A FUZZY TIME SERIES MODEL USING CAT SWARM OPTIMIZATION CLUSTERING AND PARTICLE SWARM OPTIMIZATION” by

ALHASSAN Balarabe Mohammed meets the regulations governing the award of degree of Master of Science (MSc) in Control Engineering by Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

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## DEDICATION

This research is dedicated to my parents and well-wishers.

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## ABSTRACT

This research developed a hybrid forecasting technique that integrates Cat Swarm Optimization Clustering (CSO-C) and Particle Swarm Optimization (PSO) algorithms with Fuzzy Time Series (FTS) forecasting model. Cat Swarm Optimization Clustering (CSO-C) which is an algorithm for data classification is adopted at the fuzzification stage to objectively partition the universe of discourse into unequal intervals. Then, disambiguated fuzzy relationships are obtained using Fuzzy Set Grouping (FSG). Finally, Particle Swarm Optimization (PSO) was adopted to optimize the defuzzification phase; by tuning weights assigned to fuzzy sets in a rule. This rule is a fuzzy logical relationship induced from a fuzzy set group (FSG). The clustering and optimization algorithms were implemented in MATLAB. Belgium road yearly accident data, Alabama University yearly student enrolment data, Taiwan future exchange data, University of Maiduguri (UNIMAID) yearly student enrolment data and Jigawa state yearly temperature data were collected and used to evaluate the developed hybrid model. To evaluate the forecasting efficiency of the developed hybrid model, its statistical performance metric of Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) were calculated and compared with previous techniques in the literature. Improvement was achieved in the developed forecasting technique, when compared with the benchmark Fuzzy Time Series (FTS) model of Qiang Song and Brad S. Chissom part I and II in forecasting student enrolment of University of Alabama. Results showed that an RMSE of 6.669 and MAPE result of 0.033%was obtained when compared with the benchmark work of Song and Chissom in student enrolment whose result was an RMSE of 650 and MAPE of 3.22%. There is also an improvement, in comparison to Fuzzy C- Means FTS based model of Yusuf *et al* (2015) whose result showed an RMSE of 7.02 and MAPE of 0.04%. The application of developed model on Belgium car road accident obtained an RMSE result of 5.931 and MAPE result of 0.346%which is an improvement over FCM based FTS model with RMSE of 19.2 and MAPE of 0.67%.Similarly, on application an RMSE of

2.571 and MAPE of 0.0375%were obtained in the forecast of University of Maiduguri student enrolment while in Jigawa monthly temperature forecast RMSE of 0.357 and MAPE of 0.1% were obtained. Relatively, the points on the plots followed a steady trend with the actual values for enrolment and temperature forecast respectively.

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**LIST OF ABBREVIATIONS**

|  |  |
| --- | --- |
| **Acronym** | **Definition** |
| AR | Auto-Regression |
| ARIMA | Auto-Regression Integrated Moving Average |
| CDC | Counts of Dimension of Change |
| CSO | Cat Swarm Optimization |
| CSO-C | Cat Swarm Optimization Based Clustering |
| EC | Evolutionary Computing |
| FCM | Fuzzy C-Means |
| FLC | Fuzzy Logic Control |
| FS | Fitness |
| FTS | Fuzzy Time Series |
| K | Number of Cluster |
| MA | Moving Average |
| MAPE | Mean Absolute Percentage Error |
| MdAPE | Median Absolute Percentage Error |
| MSE | Mean Square Error |

|  |  |
| --- | --- |
| PSO | Particle Swarm Optimization |
| RMSE | Root Mean Square Error |
| SMP | Seeking Memory Pool |
| SPC | Self-Position Consideration |
| SRD | Seeking Range of the Selected Dimension |
| SSE | Sum of Square Error |
| SC | Soft Computing |
| TAIFEX | Taiwan Future Exchange |

## Background

**CHAPTER ONE INTRODUCTION**

Fuzzy time series (FTS) techniques are utilized in the fields of science, engineering and general applications to develop prediction models for weather forecasting, predictive control, signal processing, population forecasting, enrolment and finance among others (Panagiotakis *et al.*, 2016).

Forecasting can be defined as the prediction of what is going to happen in the future. Researchers are of the opinion that regardless of the technique used, there can never be a perfect forecast. Meanwhile, the aim of forecasting is either to develop a prediction model that will lead to a more accurate forecasting result or an error reduced result compared to the ones in literature.

There are three classes of forecasting methods namely; qualitative, quantitative and causal (Singh, 2016). Whenever the historical data on a forecasting variable is not available or it is not applicable, the required method is referred to as qualitative forecast (Singh, 2016). This is a method that requires the judgement of an expert on that field or area to develop a forecast. On the other hand, if past information about the variable being forecasted is available and quantifiable, the required method is known as quantitative forecasting (Singh, 2016). In the latter case, forecasts are generated using time series method. The forecasting technique in which historical data is restricted to past values of the variable to be forecasted is called a time series forecasting method (Yusuf *et al.*, 2015). Causal forecasting techniques are predictions methods that are based on the assumptions that the output variable (forecast) has a cause-effect relationship with one or more variables (Anderson *et al.*, 2015).

Forecasting Techniques can further be divided into; probability theory-based (conventional) methods, computational methods, fuzzy time series and hybrid forecasting methods (Eğrioglu *et al.*, 2016).

Time series forecasting problems can be traditionally solved using linear moving average (MA) models, auto-regressive (AR) models and linear auto-regressive integrated moving average (ARIMA) models (Smith & Wunsch, 2015). Such forecasting techniques require larger observations and are unable to deal with prediction problems in which the historical data needs to be represented by linguistic values (Huang *et al.,* 2011; Shah, 2012; Song & Chissom, 1993a). Also, such techniques are confined to linearity assumptions only (Shah, 2012), which introduces large errors in the predicted values.

FTS forecasting techniques have drawn a lot of attention in recent years. However, there are certain issues associated with the development of earlier techniques (Singh, 2016) such as;

* + 1. Inaccurate determination of length of intervals.
    2. Ignorance of repeated fuzzy logic relationships.
    3. Inappropriate assignment of equal importance to fuzzy logic relationships.
    4. Utilization of first order fuzzy logic relationships.
    5. Calculation of defuzzified forecast output.

Hence, there is the need for a robust prediction technique that can uncover useful information from little historical data.

Soft Computing (SC) techniques have been utilised to deal with different challenges imposed by FTS modelling techniques (Singh, 2016). The main SC techniques for this purpose include: Artificial Neural Network (ANN), Rough Set (RS) and Evolutionary Computing (EC). Each of

them provides significant solution for addressing domain specific problems (Singh, 2016). The combination of these techniques leads to the development of a hybrid technique, which has more advantage, because it provides robust, cost effective and approximate solution, in comparison to traditional techniques. However, this combination should be computationally inexpensive and simple to implement (Singh, 2016).

Clustering techniques like K-means and fuzzy C-means have been utilized to overcome some subjective decisions made during fuzzification of FTS, such as; interval length, universe of discourse, choice of membership values, to mention but a few. These improve FTS forecasting accuracy. Cat Swarm Optimization (CSO) was developed to limit the shortcoming of premature convergence identified in the afore-mentioned clustering techniques (Chu & Tsai, 2007).

In this research CSO-C will be utilized in the fuzzification stage to objectively determine the interval length, provide objective judgement in choosing number of partitions and show good membership function between the elements in a fuzzy set. PSO will be utilized in the defuzzification stage to assign optimal weights to elements of fuzzy forecasting rules.

## Statement of Problem

The accuracy of FTS forecasting result is affected by arbitrary decisions such as static interval lengths, parametric partitioning of universe of discourse at fuzzification level, and assigning weights to recurrent fuzzy rules. It has become necessary for an FTS forecasting technique that optimizes the partitions of universe of discourse into unequal interval length, deal with recurrent fuzzy rules and assigns optimal weights to elements of a forecasting rule. As a consequence, employing CSO-C algorithm in fuzzification, Fuzzy Set Groups (FSGs) to generate logical

relationships and PSO algorithm in defuzzification will improve fuzzy time series forecasting accuracy.

## Aim and Objectives

This research aims to develop a fuzzy time series forecasting model using Cat Swarm Optimization Clustering (CSO-C) and Particle Swarm Optimization (PSO) in order to improve forecasting accuracy. The objectives of the research are as follows:

* + 1. To develop an FTS forecasting technique based on CSO-C and PSO.
    2. To apply the developed FTS forecasting technique to forecast enrolments at University of Alabama, Belgium road accident and Taiwan Future Exchange data sets.
    3. To compare the results obtained using the developed hybrid forecasting technique with results obtained using the FCM based fuzzy time series technique and to validate using university of Maiduguri enrolment data and monthly temperature data of Jigawa state.

## Scope of the Research

This work covers the development of a hybrid FTS forecasting model which empirically has the capability of forecasting a univariate data that yields improved accuracy of results using RMSE and MAPE as performance metrics. In terms of comparison, the performance of previous forecasting models with the developed model considering three standard data sets namely; Belgium car road accident data, University of Alabama student enrolment data and Taiwan future exchange (TAIFEX) data in literature were considered. Consequently, the model performance was validated using two data sets namely; UNIMAID student enrolment data and Jigawa state monthly temperature data.

## Introduction

## CHAPTER TWO LITERATURE REVIEW

This chapter has two different parts. The first part comprises the fundamental concepts relevant to the thesis and the second part provides a review of similar work.

## Review of Fundamental Concepts

This part reviews the theoretical background and fundamental concepts relevant to the context of this work.

## Time Series

Time series refers to a record that shows sequence of numerical values collected over successive period (Singh, 2016). The observation is normally spaced at uniform intervals-daily, weekly, quarterly, monthly or yearly.

It can also be defined as a sequence of well-defined observations measured at regular intervals of

time. It is a sequence of *n* data points,

*X*1, *X*2 ,..., *Xn* , consisting of continuous and non-linear

values changing with time Cheng *et al.* (2008). Time series forecasting is a mathematical method

for predicting future time series observation,

*Xn*1 , from historical time series observations,

*X*1, *X*2 ,..., *Xn* .

## Fuzzy Set Theory

This was first introduced in 1965 by Zadeh , fuzzy set theory was designed to mathematically represent and manipulate imprecise (or fuzzy) data. This was developed based on the notion of purely crisp set. In fuzzy set, elements belong to the set with a certain level of membership. It provides formal tools for dealing with uncertainty or vagueness in many problems.

**Definition 1:** Fuzzy Set; Let Y be a non-empty set and a subset of real numbers. A fuzzy set A,

in Y, the universe of discourse, is characterised by its membership function,

*A* :*Y* [0,1] . This

set A in Y is defined as a set of ordered pairs(Sets & Zadeh, 1965).

*A* {(y,

*A*  *y* ) / yY}

(2.1)

If *Y* {*y*1, *y*2 ,....*yn*}, is a finite set and A is a fuzzy set in Y then,

*A*  1 / y1  2 / *y*2  ......*n* / *yn*

(2.2)

Where *n*

is the grade membership of

*yn* and, *n* 1, 2,3.......

* + - 1. **Universe of Discourse:** The range of possible values fuzzy sets (linguistic variables) can take is called the universe of discourse. In Fuzzy Time Series (FTS), the universe of discourse,

*Y* (t), can be different at different times (Song & Chissom, 1993).

* + - 1. **Membership Function:** the membership function which represents a fuzzy set 𝑨̃ is usually denoted by 𝝁𝑨 , for an element 𝒙 of 𝑿, the value 𝝁𝑨 (𝒙) is called the membership degree of 𝒙 in the fuzzy set 𝑨̃ . The membership degree 𝝁𝑨 (𝒙) quantifies the grade of membership of the element 𝒙 to the fuzzy set 𝑨̃.

The membership function of a fuzzy set is a generalization of the indicator function in classical sets.

## Fuzzy time series

Fuzzy time series is a method of forecasting that uses linguistic mathematical reasoning to model and predict the future from a linguistic time series (Yusuf *et al.*, 2017)

The concept of fuzzy time series was first introduced by Song and Chissom (1993a).

The most important advantage of the fuzzy time series approach is to be able to work with a very small set of data.

**Definition 1**: Let *U* be the universe of discourse, where *U* {*u*1,*u*2 ,...,*un* }. then a fuzzy set A*i*

of *U* can be defined as(Bas *et al.*, 2013):

*Ai*  *A* (u1 ) / u1  *A* (u2 ) / u2 ,...,  *A* (u*n* ) / u*n*

*i i i*

(2.3)

Where 

*A*

*i*

is the membership function of the fuzzy set A*i* and *A* ;*U* [0,1]. In addition to

*A* (u *j* ), j1, 2,..., n

*i*

*i*

denote the generic elements of fuzzy set

*Ai* ; *A* (u *j* )

is the degree of

belongingness of u *j* to *Ai* ; *A* (u *j* )[0,1] .

*i*

*i*

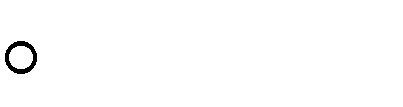
**Definition 2:** Fuzzy Time Series; let *Y* (t)(t ..., 0,1, 2,...),

a subset of real numbers, be the

universe of discourse by which fuzzy sets

*fi* (t)(i 1, 2,3,...) are defined. If F(t) is a collection of

*fi* (t)(i 1, 2,3,...), then F(t) is called a fuzzy time series defined on *Y* (t)(t 1, 2,3,...) (Yusuf *et al.*, 2015).

**Definition 3:** Fuzzy Logic Relation (FLR); if there exist a fuzzy logic relationship *R*(t1,*t*) , such that F(t)  F(t1) R(t1, t) , where represents an operator, then F(t) is said to be caused by F(t1) . The relationship between F(t) and F(t1) is denoted by;

F(t1) F(t)

(2.4)

If F(t1) A*i* and *F* (t)  A *j*

then

*Ai*  *Aj*

**Definition 4:** Fuzzy Logic Relationship Group (FLRG).

Relationships with the same fuzzy set on the left hand side can further be grouped into a relationship group. Relationship groups are also referred to as Fuzzy Logic Relationship Groups

(FLRG). Suppose that:

*Ai*  *Aj*1 , *Ai*  *Aj* 2 ,... *Ai*  *Ajn* ,

then, they can be grouped into a

relationship group as follows:

*Ai*  *Aj*1 , *Aj* 2 ,..., *Ajn* (Yusuf *et al.*, 2015).

When forecasting with Fuzzy Time Series (FTS) using real (crisp) historical data, this data must first be converted to fuzzy sets. To define fuzzy sets on the historical data, the universe of discourse, *Y* (t), the steps are as follows (Song & Chissom, 1993):

1. Find the minimum,

*D*min , and maximum,

*D*max , values of the historical data.

1. Then, define the universe of discourse, *Y* (t), as:

*Y* (*t*) [ *D*min *b*0 ,

*D*max *b*1 ]

(2.5)

where;

*b*0 and *b*1 are two positive numbers (buffers).

These buffers, according to Song and Chissom (1993), are arbitrarily assigned to adjust the lower and upper bounds of the range.

## Fuzzy Set Groups (FSGs)

In conventional Fuzzy Time Series (FTS), Fuzzy Logical Relationship Groups (FLRGs) identified, after historical data has been fuzzified, are not unique for some values. This implies that unique observations in a partition will have the same forecasted outputs which cause some mismatches between forecasts and actual historical data. These mismatches affect forecasting accuracy. Fuzzy Set Groups (FSGs) are established against Fuzzy Logical Relationship Groups (FLRGs) to give the historical data a unique set of fuzzy relations (sub patterns) which subsequently are converted to “if - then” statements. The Fuzzy Set Groups (FSGs) algorithm is implemented as follows:

Step 1: combine consecutive fuzzy sets in a pair wise manner { *F* (t 2), *F* (t1)}{A*i*,*t* 2 , A*i*,*t* 1 }

to create second – Fuzzy Set Groups (FSGs).

Step 2: if Fuzzy Set Groups (FSGs) are disambiguated, then stop; otherwise extend any ambiguous fuzzy set groups to third – order fuzzy set groups in the form

{ *F* (t 3), *F* (t 2), *F* (t1)}{A*i*,*t* 3 , A*i*,*t* 2 , A*i*,*t* 1 } to produce disambiguated fuzzy set groups. Step 3: continue the extension process until disambiguated fuzzy set groups are obtained.

Ultimately, the goal of the fuzzy set group algorithm is to obtain fuzzy relationships free of ambiguities(Poulsen, 2009). Ambiguities occur if two or more Fuzzy Set Groups (FSGs) contain the same combination of elements (Eleruja *et al.*, 2012).

## Defuzzification Operator

This is an operator that is mainly used for defuzzifying linguistic variable observations associated with the satisfaction of multiple criteria. The defuzzified output is the weighted sum

of the historical fuzzy sets values,

*ai*1 , from time *t*  *n* to *t* 1

where *n* depends on the time

series span defined by (Poulsen, 2009);

*n*

*Y* *t*   (a*t* 1w*i* )

*i*1

(2.6)

w*i* 0, 1 , is the strength of fuzzy logical relationship between past fuzzy values (inputs) and future forecasts (outputs). The closer w*i* is to 1 the stronger the relationship (Poulsen (2009).

## Basic steps of fuzzy time series forecasting

The basic steps used in fuzzy time series forecasting according to Song & Chissom (1993) are shown in the flow chart of Figure 2.1:

Fuzzify Historical Time Series Data

Start



Defuzzify Forecasted values

Performance Evaluation

End

Forecast Output Values

Estimate Fuzzy Relations

Figure 2.1: Flowchart of the Benchmark FTS Approach (Song & Chissom, 1993)

## Data Clustering

It refers to the partitioning of data or objects of the like into subclasses based on similarities. Meanwhile, objects collected in a cluster or group have something in common which is

dissimilar to the objects collected in another cluster or group. The similarity here is often defined by means of a distance norm that is measured among the data vectors themselves, or as a distance from a data vector to some prototypical object or center of the cluster (Bahrami *et al.*, 2018). Data clustering problems have been identified in many applications and domains such as computer vision and pattern recognition (video and image analysis for information retrieval, object recognition, image segmentation, and point clustering), networks (identification of web communities), databases and computing (facing privacy in databases), and statistical physics and mechanics (understanding phase transitions, vibration control, and fracture identification using acoustic emission data) (Panagiotakis *et al..*, 2016). In clustering technique, there is no information before about the number of cluster and grouping pattern (Santosa & Ningrum, 2009), that is the reason why clustering is included in unsupervised learning.

## Cat Swarm Optimization (CSO)

Cats are highly at alert and curious about their surroundings and the objects moving in their environment. In order to conserve energy, they spend most of their time resting but utilise little time on chasing preys. These behaviours help them in finding their preys and hunting them down (Bahrami *et al.*, 2018).

With inspiration of hunting pattern, Chu and Tsai (2007) developed Cat Swarm Optimization (CSO) algorithm. CSO is made up of two modes namely: “Seeking mode” which refers to the status of cats when they are resting. Secondly, the “tracing mode” that refers to the state of cats when they are chasing their prey. In the context of CSO, after creating a population of cat; the cats are randomly distributed in an M-dimension solution space, where each cat represent a solution (Bahrami *et al.*, 2018). The population of the cats are divided into two groups. While,

the cats in the first group are at rest they keep an eye on their surroundings (seeking mode). The cats in the second group start moving around to chase their preys (tracing mode).

Following Chu and Tsai (2007), the computational procedures of CSO can be described as follows:

**Phase 1**: The initial population of cats are created and disperse into the M-dimensional solution space (Xi,d) a velocity in range of the maximum velocity value (ti,d) are randomly assign to each cat.

**Phase 2**: According to the value of Mixture Ratio (MR), assign each cat a flag to sort them into the seeking or tracing mode process.

**Phase 3**: Evaluate the fitness value of each cat and save the cat with the best fitness function. The position of the best cat (Xbest) represents the best solution so far.

**Phase 4**: Based on their flags, apply the cats into the seeking or tracing mode process as described below.

**Phase 5**: If the termination criteria are satisfied, terminate the process. Otherwise repeat steps 2 through 5.

## Seeking Mode

Following Chu and Tsai (2007), the process is described below.

**Phase 1:** Make SMP copies of each cati. If the value of SPC is true, SMP-1 copies are made and the current position of the cat remains as one of the copies.

**Phase 2**: For each copy, according to CDC calculate a new position by using equation (2.6)

(Bahrami *et al.*, 2018)

*Xcn* 1 *SRD* *R* *Xc*

(2.7)

Where:

*Xc* is the current position, *X cn*

new position, and *R* a random number, which varies between 0

and 1.

**Phase 3:** Compute the fitness values (FS) for new positions. If all FS values are exactly equal, set the selecting probability to 1 for all candidate points. Otherwise calculate the selecting probability of each candidate point by using equation (2.7).

**Phase 4:** Using the roulette wheel, randomly pick the point to move to from the candidate points, and replace the position of cati.

*Pi* 

*FSi*  *FSb*

*FS*max  *FS*min

, *where* 0  *i*  *j*

(2.8)

In which:

*Pi* Is the probability of current candidate cati, *FSi*

is the fitness value of the cati,

*FS*max is the

maximum value of fitness function,

*FS*min minimum value of fitness function,

*FSb* = *FS*max

for

minimization problems and

*FSb* = *FS*min for maximization problems.

## Tracing mode (Movement)

This mode mimics a cat chasing its prey, (Chu & Tsai, 2007). Having found its prey, while resting (seeking mode), the cat decides its movement speed and direction based on the prey’s position and speed. In CSO, the velocity of cat k in dimension d is given by (Bahrami *et al.*,

2018):

*vk* ,*d* ,*new*  *vk* ,*d* ,*old*  *r*1  *c*1  *Xbest* ,*d*  *Xk* ,*d* 

(2.9)

Where;

*vk* ,*d* ,*new*

is the local position of the cat,

*Xbest* ,*d*

is the local position of catk ,

*c*1 is a constant

and *r*1 is random value in the range of [0,1].

*vk,d* = velocity of cat k in dimension d; *Xbest,d* = position of the cat with the best solution; *Xk,d* = position of the catk; *c1* = a constant; and *r1* = a random value in the range of [0,1]. With the use of aforementioned velocity, cats move in M-dimensional space that’s meant for decision. Consequently, new positions are reported. The velocity of cat is set to its maximum; if found to be greater than the maximum velocity. After which, the following equation is used to calculate new position(s) (Bahrami *et al.*, 2018).

*Xk* ,*d* ,*new*  *Xk* ,*d* ,*old*  *vk* ,*d*

(2.10)

Where, *Xk,d,new*= new position of cat k in dimension d,

*Xk,d,old*= current position of cat k in dimension d, xbest = mean value in a *cluster*

## Cat Swarm Optimization Clustering (CSO-C)

According to (Santosa & Ningrum, 2009): CSO-C is made up of two parts namely:

1. Clustering of data and
2. Searching for the best cluster center with the aid of CSO algorithm.

The following are inputs for clustering CSO:

* 1. Population of data to be used
  2. Number of clusters k
  3. Number of copy

(Santosa & Ningrum, 2009) described the phases of CSO-C as the following:

**Phase 1: Defining the initial cluster center:** In this phase, k point is chosen arbitrarily from the collected data in order to form the initial cluster center.

**Phase 2: Grouping data into clusters:** Data is imputed into cluster with the closest cluster center. Distance between data and cluster data can be obtained by (Santosa & Ningrum, 2009):



*i*1

*n*

(*x*  *y* )

2

*i*

*i*

*d* (*x*, *y*) 

*x*  *y* 

(2.11)

**Phase 3: Calculating the Sum of Squared-Error (SSE):** The fitness function of the algorithm can be obtained by:

*k*

# SSE 

( *x*  *m* 2 )

 *i*

*i*1 *x**Di*

(2.12)

Where:

*x*  *data*, *member of cluster D mi*  *cluster center i*

*k*  *number of cluster*

**Phase 4: Clustering optimization with CSO:** With regard to this algorithm, the cat is represented by a cluster center, while the new cluster center will be the solution set and is expected to come up with a smaller SSE value than before. A few adjustments are necessary in order to gain efficiency in the application of CSO to CSO-C. The adjustments are:

1. It is best to remove mixture ratio, so that every cat will have to pass the seeking and tracing mode. With this modification, it is expected that the time needed to find the best cluster center will reduce.
2. If the value of CDC were always assumed to be 100% in the seeking mode, it will allow a change for every dimension of cat copy.

**Phase 4.1: Seeking mode:** The essence of seeking mode is to search for suitable points around the cluster centres which have possibilities of becoming optimal fitness value. Hence, there is need to define three parameters namely:

* 1. Seeking Memory Pool (SMP): this will represent the number of copy a cluster have.
  2. Seeking Range of the Selected Dimension (SRD): this declares the mutative ratio, with a value between [0, 1].
  3. Self Position Considering (SPC): it is a Boolean random value (Amjad *et al.*, 2012).

The algorithm for seeking mode in CSO-C is given as follows (Santosa & Ningrum, 2009):

1. Evaluation of the parameter of seeking mode which include; SMP, SRD, SPC
2. For i = 1 to k (number of cluster center), do Copy cluster center (i) position as many as SMP.

Determine j value

Compute the shifting value (SRD\*cluster center (i))

1. For m = 1 to SMP, do

Addition or subtraction of cluster centres with shifting value is performed randomly.

\*/the output will be (SMP x k) cluster center candidates/\*

1. Compute the distance, sub classify data into clusters, and compute SSE
2. Choose a candidate to be the new cluster centre roulette wheel selection

## Phase 4.2: Updating SSE and cluster centre

Comparison is carried out between the value(s) of SSE obtained from seeking mode with the previous value of SSE; if seeking SSE value is less than earlier SSE value then the cluster centre resulting from seeking will become the new cluster centre. Conversely, if the value of seeking SSE is greater than or equal to the value of earlier SSE, we use the previous cluster centre.

**Phase 4.3: Tracing Mode:** The aim of the tracing mode is to shift point of concentration to a better position for obtaining optimal fitness value.

The Tracing Mode algorithm for CSO Clustering is as follows (Bahrami *et al.*, 2018):

1. For i = 1 to k, do Update velocity (i)

Update position (i), get the new cluster center (i)

1. Calculate the distance, grouping data into clusters, and calculate SSE

**Phase 4.4: Repeat step 4.2 for tracing SSE and cluster centre:** The value of SSE obtained from tracing mode is then compared with the previous value of SSE; if tracing SSE is less than earlier SSE then the cluster centre resulting from tracing will become the new cluster centre. Conversely, if the value of tracing SSE is greater than or equal to earlier SSE, use the previous cluster centre.

**Phase 5: Repeat phase 4 until it reaches the stopping criteria**: The complete algorithm for CSO-Clustering is shown in Figure 2.2





START

The population of data is clearly shown, show number of cluster (k), and show number of copy



If Seeking SSE is< Earlier SSE

NO

YES

Tracing SSE < Earlier SSE

NO

YES

tracing mode is Entered

seeking mode is Entered

CSO parameters are Initialized

Data is being grouped into clusters according to similarity, SSE is calculated

the initial cluster center is chosen as k data



Figure 2.2: Flowchart for CSO-C Algorithm (Santosa & Ningrum, 2009)



End

Get SSE, cluster center, and best cluster of each data

## Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is in the class of evolutionary computation (EC) and it is related to genetic algorithm and evolutionary programming, (Kennedy & Eberhart, 1999). The only thing it needs is traditional mathematical operators and in terms of speed and memory requirement it is computationally economical, (Amjad *et al.*, 2012). Empirically, it has been proven to be effective with different kinds of problems not only in the forecasting domain.

Problem optimization in PSO is achieved by having a population of candidate solution, in this process dubbed particles are moved around within a search space in accordance with simple

mathematical formula over the particles position. The particles are also guided towards the best known position in the search space and are updated as better positions are found by other particles (Amjad *et al.*, 2012).

## Performance Indices

Performance indices referred to as performance measure, are also measurement tools used to assess the accuracy or performance of developed forecasting models. In this work, Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) will be utilized as metrics for the bases of comparison between the replicated work model and developed model. Meanwhile, a smaller value of the both metric shows a sign of good forecasting (Singh, 2016).

Root Mean Square Error (RMSE); mathematically represented as (Singh, 2016):

# RMSE 

1

*n*

 *xt*  *xt* 

*T*



^ 2

*t* 1  

(2.13)

Mean Absolute Percentage Error (MAPE); also mathematically represented as:

1 *n*

*xt*  *x*

^

*MAPE*  

*n*

*x*

*t* 1 *t*

100%

(2.14)

Where;

*xt* =Actual Value

𝑥̂=Forecasted Value

*n* =Number of Forecast

## Review of Similar Work

**Song and Chissom (1993)** developed and applied a first-order, time-invariant and time variant FTS models to forecast student enrolment of the University of Alabama using linguistic value historical data. The results obtained where commendable. However, the method required large amount of computation to derive the fuzzy relation.

**Chen (1996)** presented a work simpler than the work of Song and Chissom (1993). This is because the complicated maximum minimum composition operation was replaced by a simplified arithmetic operation. However, the improved model also had the issue of recurrence number of fuzzy logical relationship group which leads to information lost.

**Huarng (2001)** work showed the importance of interval length in forecasting using fuzzy time series method. In the work, he established that to obtain effective interval lengths; the heuristic should be set in a way that at least half the fluctuation in the time series will be reflected by the chosen interval length. His work achieved better results in comparison to existing works by the effective choice of interval length. However, there was precautionary measure taken to treat the issue of recurrence number of fuzzy relationship.

**Bas *et al.* (2013)** contributed in the fuzzification and defuzzification stages. In fuzzification stage, Differential Evaluation Algorithm was utilized to avoid subjective judgments for determining the interval lengths. Discrete weights were assigned to fuzzy relation that occurred in the defuzzification process. Results obtained showed improved forecasting performance when compared with previous techniques. But, assigning of discrete weights to the recurring fuzzy relation was subjective. Thus, there is need to objectively deal with recurrence of fuzzy relation.

**Wang *et al.* (2013)** considered the effect of time variable when partitioning the universe of discourse to interpret temporal intervals. Temporal information was utilized to partition the universe of discourse into intervals with unequal lengths through Gath-Geva clustering. Results showed that the obtained intervals carry well defined semantics. Also, the experimental result showed that the partitioning with temporal information can greatly improve accuracy of forecasting students’ enrolment of Alabama University and Taiwan stock exchange capitalization weighted stock index. However, the method was not sensitive to its parameters.

**Chen and Chen (2015)** they developed a fuzzy time series forecasting model to forecast stock market prices. Their work was based on granular computing approach with binning-based partition at the fuzzification stage. The model was tested using Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX), Dow Jones Industrial Average (DJIA) and other stock index data. Their model performed better than previously existing models.

**Qiu *et al.* (2015)** presented a novel high-order fuzzy time series model based on generalized fuzzy logical relationships and automatic clustering. Experimental results showed the technique outperformed previous methods in forecasting enrolment in University of Alabama and Shanghi stock exchange. But, generalizing fuzzy logical relationships can lead to information loss.

**Bas *et al.* (2015)** proposed a new hybrid FTS forecasting method by combining high-order fuzzy-time-series forecasting model an autoregressive model. Fuzzy C-means clustering algorithm was utilized in the fuzzification of time series in a Fuzzy Time Series Network (FTSN) then the FTSN was trained by particle swarm optimization. Istanbul stock exchange daily data sets from 2009 to 2013 and the Taiwan stock exchange capitalization weighted stock index data

sets from 1999 to 2004 were used to evaluate the performance of FTSN produces more accurate forecasts for the 11 real-world time-series data sets.

**Yusuf *et al.* (2015)** developed a hybrid fuzzy time series model using fuzzy c-means at the first (fuzzification) stage to objectively partition universe of discourse into unequal interval lengths. In the model, particle swarm optimization was adopted at final stage (defuzzification) to assign optimal weights to elements of fuzzy rules. The model outperformed previous forecasting models based on its results. However it was faced with issues of handling outliers and convergence time.

**Pei (2015)** in this research, a fuzzy time series model was developed to forecast load in electric power system. The universe of discourse was unevenly partitioned using k-means algorithm. The model was based on improved fuzzification method. The results showed an improvement over previous model performance. However, there was no consideration for issue of recurrence number of fuzzy relationship

**Lu *et al.* (2015)** in the fuzzification stage, intervals were optimally obtained by continuous adjustment of width to make a more informative interval length. This means that the partitioning was based on interval information granules. The model was experimented on three datasets and better results were obtained in comparison to forecasting models previous performances. There was no explanation about dealing with recurrence number of fuzzy relation.

**Wang *et al.* (2015)** the forecasting model was a combination of a modified fuzzy c-means algorithm and granulation. It was applied to solve a time series long term prediction which was validated using data sets such as: daily temperature data, stock index data and wind speed data

among others. The experimental result showed better performance of model than other existing results from existing models. Recurrence number of fuzzy relation was not put to consideration.

**Eğrioglu *et al.* (2016)** they developed a hybrid high order fuzzy time series forecasting model. In the model, they used particle swarm optimization and feed forward neural network at the fuzzification stage and determination of fuzzy relationship stage respectively. In real time, the developed model was applied to Istanbul Stock Exchange (ISE) data. The model performed better than previous forecasting models. However, the recurrence of fuzzy relationship still remained an issue.

**Huang and Wu (2017)** they developed a hybrid fuzzy time series (FTS) model to forecast outpatient visit in which the FTS forecasting was incorporated with empirical mode decomposition to partition universe of discourse, three layer back propagation artificial neural network for the determination of fuzzy relation and particle swarm optimization to optimize the weights and threshold of bpANN. The results shown outperform the performance of previous models. However, the implementation is complex.

**Zhang *et al.* (2017)** adapted fuzzy time series model for multivariate forecasting of Shanghai Stock Exchange. Cuckoo search was utilized to partition the training data set into unequal intervals. Then relationships were generated using fuzzy logic relationship group (FLRG). Self adaptive Harmony search was utilized to integrate the secondary factor where necessary in to the forecasting rules for two variables data set. Results showed an improvement in forecasting accuracy. However computational complexities involved might make the model not work effectively for higher variable sets.

In essence, this work proposed a hybrid fuzzy time series forecasting technique to minimize the limitations of the mentioned technique; in order to improve forecasting accuracy. Cat swarm optimization algorithm will be utilized to objectively determine the interval lengths and partition the universe of discourse in the fuzzification stage. Then, Fuzzy Set Group (FSG) will be implemented to generate fuzzy relationships without recurrence. Also, particle swarm optimization will be utilized to assign optimal weights to elements of a fuzzy rule; in the defuzzification stage. CSO-C will be hybridized with PSO on FTS in order to reduce computational cost. Finally, the model will be validated using University of Alabama enrolment data, Belgium road accident data and Taiwan future exchange (TAIFEX) data. Coding the various algorithms used in the forecasting process in MATLAB will reduce computational complexity. MATLAB was chosen in this research, because of the ease to code when compared with other high-level programming languages like C++ or C#.

## Introduction

## CHAPTER THREE MATERIALS AND METHODS

This chapter is focused on the discussion of steps of the methodology and their implementations. The implementation of the methodology is a detailed explanation of the steps itemized in achieving the objectives.

## Materials

The materials utilised in carrying out this work include; Matrix Laboratory application software; MATLAB 2016a, five historical data sets; obtained from the benchmark FTS work of Song and Chissom in 1993 part II, and historical data sets of student enrolment data of Alabama University. Also obtained are Belgium car road accident data from the work of Yusuf *et al.* (2015), Taiwan Future Exchange (TAIFEX) data from the work of Bas *et al.* (2015), and student enrolment data for University of Maiduguri which was obtained from the Academic Planning unit of the University. The time series has 18 yearly enrolments between 1976 and 1993. The data can be found in Appendix D and monthly temperature data for Jigawa State, Laptop computer with Intel Core (TM) i3-3250M micro processor, frequency speed rate (2.30GHz) and Random Access Memory (RAM) of 4.00GB, is also utilized. All the data used are shown in the appendices.

## Methods

This section focuses on discussing steps of the methodology and their implementation. The implementation of the methods is a detailed explanation of the steps itemized in the methodology which include:

1. Development of an FTS forecasting technique based on CSO-C and PSO.
   1. Code the fuzzification module, based on CSO-C in MATLAB.
   2. Generate disambiguated fuzzy relationship using FSG.
   3. Convert fuzzy relationship to “if-then” rules.
   4. Code the defuzzification module, based on PSO, in MATLAB.
   5. Optimize the elements of the “if-then” rules using the PSO algorithm coded in MATLAB.
   6. Generate forecasts.
2. Application of the developed FTS technique to forecast enrolment at University of Alabama, Belgium Car Road Accident, Taiwan Future Exchange (TAIFEX), enrolment of University of Maiduguri, monthly temperature of Jigawa state.
3. Collection and processing of data sets.
4. Apply each data set on (a) to compute partitions and membership degree.
5. Fuzzify data set using ordered partitions and membership degree.
6. Application of 1(b-e).
7. Generate forecasts for data sets.
8. Comparison of the results obtained from the developed FTS technique with that obtained using the previous techniques or methods; with the use of RMSE and MAPE as performance metric.

## Development of an FTS forecasting technique based on CSO-C and PSO.

The aim of integrating Cat Swarm Optimization Clustering (CSO-C) into the fuzzification stage of FTS is to determine empirically the partitions of interval lengths and generate membership values for any type of data sets. This eliminates the need to define universe of discourse and

generates optimal partitions. The algorithm for CSO-C implemented in this work is shown in figure 3.1 below:



Start

Get Training data set

Set the CSO-C

Compute cluster centres, mi

Get mi, 

Compute membership values,

Stop

Compute objective function, J

No

Is J < ε

Yes

Figure 3.1: Flow Chat for the Fuzzification Module

The flow chat in Figure 3.1 is an illustration of the processes carried out at the fuzzification module of the proposed FTS model. The flow chart of “the computation of cluster centres done by CSO-C” is as shown in Figure 2.2.

The CSO-C parameters are then set. These parameters include Seeking Memory Pool (SMP), Seeking Range of The Selected Dimension (SRD), and Self-Position Considering (SPC). SMP represents how many copy a cluster center has. SRD declares the mutative ratio, with a value between [0, 1]. SPC is a Boolean random value. Table 3.1 shows the CSO-C parameters and their respective specifications.

Table 3.1: Showing CSO-C parameters and specifications (Santosa & Ningrum, 2009)

|  |  |
| --- | --- |
| **Parameters** | **Specifications** |
| SMP CDC SRD  Const1 r1 Velmax  Number of clusters  Maximum number of  iterations | 5  100%  0.2  2  [0,1]  0.9  7  100 |

The following Table shows the cluster centers obtained for all the data sets: Table 3.2: Cluster Centers Obtained for all Data Sets

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Cluster Centers | Car Road Accident  Data | Alabama Uni.Enrolment  Data | TAIFEX  Data | UNIMAID  Enrolment Data | Jigawa Temperature  Data |
| m1 | 1172.10 | 13055.11 | 6709.75 | 743.00 | 33.20 |
| m2 | 1380.00 | 13565.35 | 6806.00 | 2925.00 | 33.70 |
| m3 | 1432.00 | 15164.65 | 6871.00 | 5800.00 | 35.50 |
| m4 | 1478.10 | 15862.01 | 6890.00 | 7238.00 | 36.90 |
| m5 | 1574.06 | 16917.99 | 6926.00 | 7687.00 | 37.50 |
| m6 | 1616.00 | 18149.95 | 6952.75 | 9884.00 | 37.70 |
| m7 | 1644.00 | 19333.69 | 7039.00 | 11410.00 | 38.10 |

As seen from Table 3.2 above that the cluster centers are represented in ascending order in all the data sets.

Next is to define fuzzy sets from partitions generated using membership degree. Fuzzy Set Groups are established against the conventional Fuzzy Logic Relationship Groups (FLRGs) to

deal with recurrence of fuzzy relationships. Subsequently, each FSG is converted into unique “if- then” statements. The snippet for “the establishment of disambiguated FLR” is as follows:

*% Step 3.2: Obtain FSG FSG = fuzzySetGroup(F);*

“if-then” rules are generated on the bases of the element in the FSG using equation (3.1) below (Yusuf *et al.*, 2015)

*if* (F(t1)  A*r* ,*t* 1  F(t 2)  A*r* ,*t* 2  ...  *F* (t n 1) 

A*r* ,*t* *n*1  *F* (t n)  A*r* ,*t* *n* )

(3.1)

then,

w*t* 1  ? w*t* 2  ?... w*t* *n*1  ? w*t* *n*  ?

(3.2)

Where;

w*t*1 = the weight of the previous historical data point at time (t-n)

t = time (period) of the previous historical data point whose forecast is required.

n = time (period) of the previous historical data point matched in a forecasting rule.

This weight w*t**n* , represents the strength of fuzzy logical relationship between the previous historical data at n and future forecast at t.

Subsequently, the “if-then” rule is tuned using Particle Swarm Optimization (PSO) integrated into the defuzzification phase. PSO algorithm is also implemented in MATLAB 2016. The snippet for the PSO algorithm is shown as follows:

*R = (Panagiotakis et al., 2016); for i=1:size(D,1)*

*R{i} = [];*

*fuzzySet = FSG(i, find(FSG(i,:)>0)); fuzzySet = fuzzySet(end:-1:1);*

*if ~isempty(fuzzySet)*

*mj = umid(fuzzySet)'; ub = ones(size(mj)); % lb = zeros(size(mj));*

*A = D(i,2);*

*nvars = numel(mj);*

*fun = @(r) (sum(r.\*mj) - A)^2;*

*r = particleswarm(fun,nvars,lb,ub); R{i} = r;*

*x\_hat(i,:) = sum(mj .\* r); end*

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Set loop counter *k = k + 1*

Start

Get and set fuzzy sets

Set fuzzy rules

Update current velocity, *Vi*

Update current particle position, *Xi(current)*

Set PSO parameters

*Kmax*, ε, *c1*,*c2*, ω and *n*

Compute Current SSEs, *Jnew*

No

Initialize particles

position *Xn,i(initial)*

Jnew(min)< Jint ?

Compute Initial SSEs, *Jn*

Yes

Update Old (intermediate) SSE, Jint

Update personal best particles and global best particle

No Jnew(min)<

ε?

Yes



Stop

Figure 3.2: Flowchart of the Particle Swarm Optimization Algorithm (Yusuf *et al.*, 2015)

Get global best particle

The main PSO parameters applied to produce optimal solutions are; swarm size, maximum iteration, learning factors, particles initial positions, inertial weight factor and target fitness value (Yusuf *et al.*, 2015). Table 3.7 shows the set values of the PSO parameters.

Table 3.3: The PSO Parameters (Yusuf *et al.*, 2015)

|  |  |
| --- | --- |
| **Parameters** | **Specifications** |
| Swarm Size  Maximum Number of Iterations Target Fitness Value as MSE  Min and Max Particles Position Limited to Min. and Max. Vel. Range  Learning Factors C1 and C2 Inertial Coefficient, w  Maximum number of iterations | 5  500  1  [0,1]  [-0.01,0.01]  2  1.4  100 |

The overall FTS algorithm of the complete CSO-C code developed is shown in Appendix Q.

## Application of the Developed FTS Technique to Forecast Data

In this subsection, the application of the proposed technique to forecasting five data sets is discussed. These data sets include: Belgium car road accident data, Alabama University student enrolment data, Taiwan future exchange data, Maiduguri University student enrolment data and Jigawa state monthly temperature data. The numbers of clusters were set to seven; this is due to the fact that small numbers of partitions affect forecasting rule and accuracy. While, large numbers of partitions diminish the use of fuzzy time series by not allowing adequate number of fluctuations in the process. The maximum number of iteration is set to one hundred as a stopping

criterion. In each case the code is run thirty times after which the best of all the obtained result is chosen. The tables are provided in Appendix V to Z.

## Forecasting Car Road Accident in Belgium

Forecasting car road accident in Belgium, involves converting the data to linguistic values using defined fuzzy sets and membership degree of each data point computed. The CSO-C algorithm is coded in such a way that one of its outputs is a collection of data points that belong to a cluster centre, based on their maximum degree. Table 3.4 shows the fuzzified training data set for car road accident.

Table 3.4: Fuzzification of Car Road Accident Data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Date** | **Training Data Set** | **Fuzzy Set** | **Date** | **Training Data Set** | **Fuzzy Set** |
| 1975 | 1460 | A4 | 1990 | 1574 | A5 |
| 1976 | 1536 | A5 | 1991 | 1471 | A4 |
| 1977 | 1597 | A6 | 1992 | 1380 | A2 |
| 1978 | 1644 | A6 | 1993 | 1346 | A2 |
| 1979 | 1572 | A6 | 1994 | 1415 | A3 |
| 1980 | 1616 | A7 | 1995 | 1228 | A1 |
| 1981 | 1564 | A6 | 1996 | 1122 | A1 |
| 1982 | 1464 | A4 | 1997 | 1150 | A1 |
| 1983 | 1479 | A3 | 1998 | 1224 | A1 |
| 1984 | 1369 | A3 | 1999 | 1173 | A1 |
| 1985 | 1308 | A2 | 2000 | 1253 | A1 |
| 1986 | 1456 | A4 | 2001 | 1288 | A2 |
| 1987 | 1390 | A2 | 2002 | 1145 | A1 |
| 1988 | 1432 | A3 | 2003 | 1035 | A1 |
| 1989 | 1488 | A4 | 2004 | 953 | A1 |

The next step is the generation of fuzzy rules using FSG. Table 3.9 shows the generated fuzzy rule during the first pass.

Table 3.5: First Pass in Generating Fuzzy Rules for Car Road Accident Data

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Data Points** | **Date** | **Fuzzy Set** | **Rule** | **Data Points** | **Date** | **Fuzzy Set** | **Rule** |
| 1 | 1975 | A4 | #, # | 16 | 1990 | A5 | A5, A5 |
| 2 | 1976 | A5 | #,A5 | 17 | 1991 | A4 | A5 ,A6 |
| 3 | 1977 | A6 | A5 , A6 | 18 | 1992 | A2 | A6, A5 |
| 4 | 1978 | A6 | A6, A7 | 19 | 1993 | A2 | A5,A4 |
| 5 | 1979 | A6 | A7 , A7 | 20 | 1994 | A3 | A4,A4 |
| 6 | 1980 | A7 | A7 , A6 | 21 | 1995 | A1 | A4,A5 |
| 7 | 1981 | A6 | A6 , A7 | 22 | 1996 | A1 | A5, A3 |
| 8 | 1982 | A4 | A7, A6 | 23 | 1997 | A1 | A3, A1 |
| 9 | 1983 | A3 | A6, A5 | 24 | 1998 | A1 | A1, A2 |
| 10 | 1984 | A3 | A5, A5 | 25 | 1999 | A1 | A2, A3 |
| 11 | 1985 | A2 | A5 , A4 | 26 | 2000 | A1 | A3, A2 |
| 12 | 1986 | A4 | A4, A4 | 27 | 2001 | A2 | A2, A3 |
| 13 | 1987 | A2 | A4, A5 | 28 | 2002 | A1 | A3, A4 |
| 14 | 1988 | A3 | A5, A4 | 29 | 2003 | A1 | A4, A1 |
| 15 | 1989 | A4 | A4 , A5 | 30 | 2004 | A1 | A1,A1 |

It can be seen from Table 3.5 that not all fuzzy set groups are unique. Ambiguity occurs for the years (1977-1979); data points 3 through 5, 1981; data point 7, (1983-1984, 1988); data points 9,

10 and 14, (1985 and 1987); data points, (1975, 1982, 1986, 1989, 1991); data points 1, 8, 12, 15

and 17, (1976, 1990); data points 16 and 2, (1992, 1993 and 2001); data points 18, 19 and 27, which is also the same with data points 11 and 13 that is (1985,1987). We also have (1995-2000 and 2002-2004); data points 21 through 26 and 28 through 30.

In order to obtain disambiguated fuzzy rule, the ambiguous fuzzy set groups are extended to the next order, and so on, until unique groups are established, Yusuf *et al.* (2015). This extension is

achieved by adding the previous available linguistic observation until a higher order fuzzy relationship is attained. Table 3.6 shows the disambiguated fuzzy relationships for car road accident in Belgium.

Table 3.6: Fuzzy Set Group and Respective Optimal Weights for Car Accident Data

|  |  |  |
| --- | --- | --- |
| **Data points** | **Maps** | **Optimal weight(s)** |
| 1 | #, #→ A4 | #,# |
| 2 | #,A5→ A5 | #,# |
| 3 | A5, A6→ A6 | 0.022442, 0.977558 |
| 4 | A5, A6, A7→ A6 | 0.23847,0.0023311, 0.81188 |
| 5 | A7, A7→ A6 | 0.98302, 0 |
| 6 | A7, A7 , A6→A7 | 0.47619, 0.28692, 0.25285 |
| 7 | A7, A6, A7→ A6 | 0.45287, 0.08873, 0.43831 |
| 8 | A6, A7, A6→ A4 | 0.93558, 0, 0 |
| 9 | A7, A6, A5→ A3 | 0.081525,0.041537, 0.89499 |
| 10 | A6,A5, A5→ A3 | 0.47014, 0.19109, 0.25047 |
| 11 | A5, A5, A4→ A2 | 0.34784, 0.14902, 0.44225 |
| 12 | A5,A5, A4, A4→ A4 | 0.983841, 0, 0, 0.016159 |
| 13 | A5,A5,A4, A4, A5 →A2 | 0.42501,0, 0.5797, 0,0 |
| 14 | A4, A5, A4→ A3 | 0.21641, 0.7953, 0 |
| 15 | A5, A4, A5→ A4 | 0, 0.076319, 0.96584 |
| 16 | A4, A5, A5→ A5 | 0.46637, 0.40469, 0.25499 |
| 17 | A5, A5 ,A6→ A4 | 0.54514, 0.24358, 0.21717 |
| 18 | A5, A6, A5 → A2 | 0.69881, 0, 0.26337 |
| 19 | A6,A5,A4→ A2 | 0 , 0 |
| 20 | A6,A5, A4,A4→A3 | 0.32258,0.42598,0.13994,0.082344 |
| 21 | A6,A5,A4,A4,A5→A1 | 0.0073059,0, 0.034287, 0.86937 |
| 22 | A5, A3→ A1 | 0.15219, 0.73786 |
| 23 | A3, A1→A1 | 0.72378, 0.23485 |
| 24 | A1, A2→ A1 | 0.044945, 0.955055 |
| 25 | A1, A2, A3→ A1 | 0.33401, 0.68015, 0 |
| 26 | A3, A2→A1 | 0.99895, 0.025133 |
| 27 | A3, A2, A3→ A2 | 0, 0.13372, 0.92348 |
| 28 | A3, A4→A1 | 0.93486, 0 |
| 29 | A4, A1→A1 | 0.34751, 0.50543 |
| 30 | A1,A1→A1 | 0.023115, 0.82616 |

As seen in table 3.6 the fuzzy relationship for the first two data points cannot be established since FSG approach requires a minimum of two previous linguistic observations to match the current linguistic observation. Hence, no weights are assigned to such fuzzy rules.

## Forecasting Students Enrolments in University of Alabama

Similarly, forecasting student enrolments in University of Alabama involves converting the data to linguistic values using defined fuzzy sets and membership degree of each data point computed. Table 3.7 shows the fuzzified training data set for student enrolments in University of Alabama.

Table 3.7: Fuzzification of Alabama Student Enrolment Data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Date** | **Training**  **Data Set** | **Fuzzy Set** | **Date** | **Training**  **Data Set** | **Fuzzy Set** |
| 1971 | 13055 | A1 | 1982 | 15433 | A3 |
| 1972 | 13563 | A2 | 1983 | 15497 | A3 |
| 1973 | 13867 | A2 | 1984 | 15145 | A3 |
| 1974 | 14696 | A3 | 1985 | 15163 | A3 |
| 1975 | 15460 | A3 | 1986 | 15984 | A4 |
| 1976 | 15311 | A3 | 1987 | 16859 | A5 |
| 1977 | 15603 | A4 | 1988 | 18150 | A6 |
| 1978 | 15861 | A4 | 1989 | 18970 | A7 |
| 1979 | 16807 | A5 | 1990 | 19328 | A7 |
| 1980 | 16919 | A5 | 1991 | 19337 | A7 |
| 1981 | 16388 | A5 | 1992 | 18876 | A7 |

Then, the linguistic observations are utilized in generating fuzzy rules. Table 3.8 shows the generated fuzzy rules during the first pass for Alabama university student enrolment.

Table 3.8: First Pass in Generating Fuzzy Rules for Alabama University Student Enrolment Data

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Data**  **Points** | **Date** | **Fuzzy**  **Sets** | **Rule** | **Data**  **Points** | **Date** | **Fuzzy**  **Sets** | **Rule** |
| 1 | 1971 | A1 | #, # | 12 | 1982 | A3 | A6, A5 |
| 2 | 1972 | A2 | #, A1 | 13 | 1983 | A3 | A5, A2 |
| 3 | 1973 | A2 | A1 , A1 | 14 | 1984 | A3 | A2, A2 |
| 4 | 1974 | A3 | A1, A1 | 15 | 1985 | A3 | A2, A2 |
| 5 | 1975 | A3 | A1, A2 | 16 | 1986 | A4 | A2, A2 |
| 6 | 1976 | A3 | A2, A2 | 17 | 1987 | A5 | A2, A4 |
| 7 | 1977 | A4 | A2, A2 | 18 | 1988 | A6 | A4, A6 |
| 8 | 1978 | A4 | A2 , A3 | 19 | 1989 | A7 | A6, A7 |
| 9 | 1979 | A5 | A3, A4 | 20 | 1990 | A7 | A7, A7 |
| 10 | 1980 | A5 | A4, A6 | 21 | 1991 | A7 | A7, A7 |
| 11 | 1981 | A5 | A6, A6 | 22 | 1992 | A7 | A7, A7 |

As seen in table 3.8 there is occurrence of ambiguity in the years: (1972, 1973); data points 2 and 3, (1974-1976 and 1982-1985); data points 4 through 6 and data points 12 through 15, (1977,

1978, and 1986); data points 7, 8 and 16, (1979, 1980, 1981 and 1987); data points 9, 10, 11 and 17, (1989-1992); data points 19 through 22. The historical observations of the university student enrolment data are converted to linguistic values using defined fuzzy sets and membership degree of each data point computed. This is a common challenge caused by the nature of data set. The solution to avoiding such information loss is to choose a very robust defuzzification approach as implemented in this work. This implies that FLRG is not suitable. Hence, there is need to utilize FSG or disambiguated fuzzy relation. Table 3.9 shows the disambiguated fuzzy relationship for Alabama University student enrolment.

Table 3.9: Fuzzy Set Group and Respective Optimal Weights for Alabama Student Enrolment Data

|  |  |  |
| --- | --- | --- |
| **Data**  **Points** | **Maps** | **Optimal Weight(S)** |
| 1 | #, #→ A1 | #, # |
| 2 | #, A1 → A2 | #, # |
| 3 | A1 , A1 → A2 | 0 , 0.955055 |
| 4 | A1, A1, A1→A3 | 0.85229, 0, 0.20749 |
| 5 | A1, A2→A3 | 0, 0.56155 |
| 6 | A1, A2, A2→A3 | 0, 0.42887, 0.56148 |
| 7 | A1, A2, A2, A2→A4 | 0.9149, 0, 0, 0.18862 |
| 8 | A2 , A3→A4 | 0.9743, 0.0257 |
| 9 | A3, A4→A5 | 0.052746, 0.947254 |
| 10 | A3, A4, A6→A5 | 0, 0, 0.9743 |
| 11 | A6, A6→A5 | 0.21978, 0.74883 |
| 12 | A6, A5→A3 | 0.14587, 0.79113 |
| 13 | A5, A2→A3 | 0, 0.18867 |
| 14 | A5, A2, A2→A3 | 0, 0.37964, 0.59998 |
| 15 | A5, A2, A2, A2→A3 | 0.80891, 0, 0 |
| 16 | A2, A2, A2, A2→A4 | 0, 0.033894, 0 |
| 17 | A2, A4→ A5 | 0.95174, 0.13425 |
| 18 | A2, A4, A6→A6 | 0.090562,0.28332,0.72234 |
| 19 | A6, A7→A7 | 0.090562,0.28332,0.72234 |
| 20 | A6, A7, A7→A7 | 0.02116, 0, 0.97884 |
| 21 | A6, A7, A7, A7→A7 | 1,0, 0.085242, 0.042223 |
| 22 | A7, A7, A7, A7→A7 | 0, 0, 0.41191, 0.58313 |

## Forecasting Taiwan future exchange data (TAIFEX)

Forecasting Taiwan future exchange data is also done in similar manner Belgium car road accident and Alabama University student enrolment is carried out. Table 3.10 shows the fuzzified training data set for TAIFEX data.

Table 3.10: Fuzzification of TAIFEX Data.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Date** | **Training Data Set** | **Fuzzy Set** | **Date** | **Training Data Set** | **Fuzzy Set** |
| 1 | 7552 | A1 | 9 | 6861 | A3 |
| 2 | 6726.5 | A1 | 10 | 6926 | A5 |
| 3 | 6774.55 | A2 | 11 | 6852 | A3 |
| 4 | 6762 | A2 | 12 | 6890 | A4 |
| 5 | 6952.75 | A6 | 13 | 6871 | A3 |
| 6 | 6906 | A5 | 14 | 6840 | A3 |
| 7 | 6842 | A3 | 15 | 6806 | A2 |
| 8 | 7039 | A7 | 16 | 6787 | A2 |

The fuzzified dataset is then utilized to generate fuzzy rules. Table 3.11 shows the first pass for generating fuzzy rules for TAIFEX data.

Table 3.11: First Pass in Generating Fuzzy Rules for TAIFEX Data

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Data Points** | **Date** | **Fuzzy Sets** | **Rule** | **Data Points** | **Date** | **Fuzzy Sets** | **Rule** |
| 1 | 03.08.1998 | A1 | #, # | 9 | 21.09.1998 | A3 | A4, A7 |
| 2 | 11.09.1998 | A1 | #, A1 | 10 | 22.09.1998 | A5 | A7, A4 |
| 3 | 12.09.1998 | A2 | A1,A1 | 11 | 23.09.1998 | A3 | A4, A5 |
| 4 | 15.09.1998 | A2 | A1, A3 | 12 | 24.09.1998 | A4 | A6, A4 |
| 5 | 16.09.1998 | A6 | A3,A2 | 13 | 25.09.1998 | A3 | A4,A5 |
| 6 | 17.09.1998 | A5 | A2, A6 | 14 | 28.09.1998 | A3 | A5, A4 |
| 7 | 18.09.1998 | A3 | A6, A5 | 15 | 29.09.1998 | A2 | A4,A4 |
| 8 | 19.09.1998 | A7 | A5, A4 | 16 | 30.09.1998 | A2 | A4,A3 |

Table 3.15 shows the occurrence of ambiguity on the following dates: (03.08.98, 11.09.98); data points 1 and 2, (12.09.98, 15.09.98); data points 3 and 4, (18.09.98, 21.09.98, 23.09.98, 25.09.98,

and 28.09.98); data points 7, 9, 11, 13 and14, (17.09.98, 22.09.98); data points 6 and 10. Table

3.12 shows the rule generated and weights assigned using FSG and PSO, respectively.

Table 3.12: Fuzzy Set Group and Respective Optimal Weights for TAIFEX Data

|  |  |  |
| --- | --- | --- |
| **Data**  **Points** | **Maps** | **Optimal**  **Weight(S)** |
| 1 | #, #→ A1 | #, # |
| 2 | #, A1 → A1 | #, # |
| 3 | A1,A1→A2 | 0.024726, 0.98242 |
| 4 | A1, A3→A2 | 0, 0.99808 |
| 5 | A3,A2→A6 | 0.074917, 0.95315 |
| 6 | A2, A6→A5 | 0.54409, 0.4641 |
| 7 | A6, A5→A3 | 0.28698, 0.70344 |
| 8 | A6, A5, A4→A7 | 0.987482, 0.012518, 0 |
| 9 | A4, A7→A3 | 0.99808, 0 |
| 10 | A7, A4 → A5 | 0.0092343, 0.987482 |
| 11 | A4, A5→A3 | 0.0987651,0 |
| 12 | A6, A4→A4 | 0.004171, 0.995829 |
| 13 | A4,A5→A3 | 0.99066, 0.010751 |
| 14 | A4, A5, A4→A3 | 0.42951, 0.56504, 0 |
| 15 | A4,A4→A2 | 0.99198,0 |
| 16 | A4,A3→A2 | 0.0017454, 0.995829 |

Table 3.12 shows the disambiguated fuzzy relationships and the weights assigned for the TAIFEX data set.

## Forecasting Student Enrolments in University of Maiduguri

Here, forecasting student enrolments in University of Maiduguri (UNIMAID) data from 1976 to 1993 is considered. The obtained data is shown in Appendix D. As seen there are no missing observations for the range of years. Table 3.13 shows fuzzified training dataset for UNIMAID student enrolment data.

Table 3.13: Fuzzified Data for UNIMAID

|  |  |  |
| --- | --- | --- |
| **Date** | **Training Data Set** | **Fuzzy Set** |
| 1976 | 743 | A1 |
| 1977 | 1128 | A1 |
| 1978 | 1882 | A2 |
| 1979 | 2500 | A2 |
| 1980 | 2925 | A2 |
| 1981 | 3251 | A2 |
| 1982 | 4561 | A3 |
| 1983 | 5329 | A3 |
| 1984 | 5719 | A3 |
| 1985 | 5800 | A3 |
| 1986 | 6168 | A3 |
| 1987 | 6711 | A4 |
| 1988 | 7238 | A4 |
| 1989 | 7687 | A5 |
| 1990 | 7960 | A5 |
| 1991 | 8302 | A5 |
| 1992 | 9884 | A6 |
| 1993 | 11410 | A7 |

Table 3.14 shows the first pass of generating fuzzy rule for UNIMAID data.

Table 3.14: First Pass in Generating Fuzzy Rules for UNIMAID Student Enrolment Data

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Data Points** | **Date** | **Fuzzy Sets** | **Rule** | **Data Points** | **Date** | **Fuzzy Sets** | **Rule** |
| 1 | 1976 | A1 | #,# | 10 | 1985 | A3 | A3, A3 |
| 2 | 1977 | A1 | #, A1 | 11 | 1986 | A3 | A3, A3 |
| 3 | 1978 | A2 | A1, A1 | 12 | 1987 | A4 | A3, A3 |
| 4 | 1979 | A2 | A1, A2 | 13 | 1988 | A4 | A3, A4 |
| 5 | 1980 | A2 | A2, A2 | 14 | 1989 | A5 | A4, A4 |
| 6 | 1981 | A2 | A2, A2 | 15 | 1990 | A5 | A5, A5 |
| 7 | 1982 | A3 | A2, A2 | 16 | 1991 | A5 | A5, A5 |
| 8 | 1983 | A3 | A2, A3 | 17 | 1992 | A6 | A5,A5 |
| 9 | 1984 | A3 | A3, A3 | 18 | 1993 | A7 | A5,A6 |

It can be seen from table 3.14 that the fuzzy set groups that are not unique include: (1976, 1977); data points 1 and 2, (1978-1981); data points 3 through 6, (1982-1986); data pints 7 through 11,

(1987, and 1988); data points 12 and 13, (1989-1991); data points 14, 15 and 16.

Table 3.15: Fuzzy Set Group and Respective Optimal Weights for UNIMAID Data

|  |  |  |
| --- | --- | --- |
| **Data Points** | **Maps** | **Optimal Weight(S)** |
| 1 | #,# → A1 | #, # |
| 2 | #, A1 → A1 | #, # |
| 3 | A1, A1→ A2 | 0.654322, 0.345678 |
| 4 | A1, A2→ A2 | 0.54268,0.71685 |
| 5 | A1, A2, A2→A2 | 0, 0.654322, 0.457320 |
| 6 | A1, A2, A2, A2 → A2 | 0.0076546,0.654382,0.10951,0.54268, |
| 7 | A2, A2, A2, A2 → A3 | 0.013258,0.45732,0.653578,0.73428 |
| 8 | A2, A3 → A3 | 0.8431, 0.49361 |
| 9 | A2,A3, A3→ A3 | 0.456781 , 0.322459 , 0.98603 |
| 10 | A2, A3, A3, A3 → A3 | 0.875324,0.555671, 0.234500,0.53987 |
| 11 | A2, A3, A3, A3, A3→ A3 | 0,0.12576,0.025119,0.232678,0.45530 |
| 12 | A3, A3, A3, A3, A3 → A4 | 0.288830, 0, 0.31378, 0.55446, 0.984230 |
| 13 | A3, A4 → A4 | 0, 0.4563210 |
| 14 | A4, A4→A5 | 0.28253, 0.779500 |
| 15 | A4, A5, A5 → A5 | 0.037718, 0.675431 |
| 16 | A4, A5, A5→ A5 | 0.074139, 0.026097, 0.984100 |
| 17 | A5, A5,A5→ A6 | 0.86899, 0.18538, 0.23143 |
| 18 | A5,A6→ A7 | 0.6744321, 0.3766734 |

Table 3.15 shows the generated rule and assigned weight using FSG for UNIMAID student enrolment dataset.

## Forecasting Monthly Temperature of Jigawa state

Forecasting monthly temperature data of Jigawa state from 1982 to 2013 is considered in this subsection. Table 3.16 shows the fuzzified temperature data for Jigawa state, for the month of July.

Table 3.16: Fuzzified Monthly Temperature Data for Jigawa (July)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Date** | **Training Data Set** | **Fuzzy Set** | **Date** | **Training Data Set** | **Fuzzy Set** |
| 1982 | 36.2 | A1 | 1998 | 36.9 | A7 |
| 1983 | 34.8 | A1 | 1999 | 37.8 | A4 |
| 1984 | 36.9 | A2 | 2000 | 38.4 | A5 |
| 1985 | 35.6 | A2 | 2001 | 36.7 | A5 |
| 1986 | 36.4 | A2 | 2002 | 38.9 | A5 |
| 1987 | 33.7 | A2 | 2003 | 38.1 | A5 |
| 1988 | 35.0 | A3 | 2004 | 38 | A6 |
| 1989 | 35.3 | A3 | 2005 | 36.7 | A2 |
| 1990 | 36.3 | A3 | 2006 | 39.5 | A2 |
| 1991 | 37.0 | A4 | 2007 | 37.3 | A3 |
| 1992 | 35.5 | A4 | 2008 | 36.6 | A3 |
| 1993 | 35.2 | A4 | 2009 | 36.9 | A3 |
| 1994 | 35.8 | A5 | 2010 | 37.4 | A3 |
| 1995 | 34.5 | A4 | 2011 | 37.7 | A3 |
| 1996 | 32.4 | A5 | 2012 | 36.1 | A5 |
| 1997 | 33.2 | A5 | 2013 | 37.5 | A6 |

Table 3.17 shows the first pass of generating fuzzy rule for monthly temperature data of Jigawa state.

Table 3.17: First Pass in Generating Fuzzy Rules for Jigawa State Monthly Temperature Data

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **DATA POINTS** | **DATE** | **FUZZY SETS** | **RULE** | **DATA POINTS** | **DATE** | **FUZZY SETS** | **RULE** |
| 1 | 1982 | A1 | #,# | 17 | 1998 | A4 | A1, A1 |
| 2 | 1983 | A1 | #, A1 | 18 | 1999 | A6 | A1, A4 |
| 3 | 1984 | A4 | A3, A3 | 19 | 2000 | A7 | A4, A6 |
| 4 | 1985 | A3 | A3, A4 | 20 | 2001 | A4 | A6, A7 |
| 5 | 1986 | A4 | A4, A3 | 21 | 2002 | A7 | A7, A4 |
| 6 | 1987 | A2 | A3,A4 | 22 | 2003 | A7 | A4, A7 |
| 7 | 1988 | A3 | A4, A2 | 23 | 2004 | A7 | A7 ,A7 |
| 8 | 1989 | A3 | A2, A3 | 24 | 2005 | A4 | A7, A7 |
| 9 | 1990 | A4 | A3, A3 | 25 | 2006 | A7 | A7, A4 |
| 10 | 1991 | A4 | A3 ,A4 | 26 | 2007 | A5 | A4, A7 |
| 11 | 1992 | A3 | A4, A4 | 27 | 2008 | A4 | A7, A5 |
| 12 | 1993 | A3 | A4, A3 | 28 | 2009 | A4 | A5, A4 |
| 13 | 1994 | A3 | A3, A3 | 29 | 2010 | A5 | A4, A4 |
| 14 | 1995 | A2 | A3, A3 | 30 | 2011 | A6 | A4, A5 |
| 15 | 1996 | A1 | A3, A2 | 31 | 2012 | A3 | A5, A6 |
| 16 | 1997 | A1 | A2, A1 | 32 | 2013 | A5 | A6, A3 |

It can be seen from Table 3.17 that, there is ambiguity in (1982, 1983, 1996 and 1997); data points 1, 2, 15 and 16, (1987, 1995); data points 6 and 14, (1985, 1988, 1989, 1992, 1993, 1994

and 2012); data points 4, 7, 8, 11, 12, 13 and 31, (1984, 1986, 1990, 1991, 1998, 2001, 2005,

2008 and 2009); data points 3, 5, 9, 10, 17, 20, 24, 27 and 28, (2007, 2010 and 2013); data points

26, 29, 32, (1999, 2011); data points 18 and 30, (2000, 2002, 2003, 2004 and 2006); data points

19, 21, 22, 23 and 25. Table 3.18 shows the assigned weights and rules generated using FSG and PSO, respectively for Jigawa Monthly Temperature Data.

Table 3.18: Fuzzy Set Group and Respective Optimal Weights for Jigawa Monthly Temperature Data

|  |  |  |
| --- | --- | --- |
| **Data Points** | **Maps** | **Optimal Weight(S)** |
| 1 | #,# →A1 | #, # |
| 2 | #, A1 → A1 | #, # |
| 3 | A3, A3 → A4 | 0.96888, 0.070553 |
| 4 | A3, A3, A4 → A3 | 0.02464, 0.22839, 0.00271 |
| 5 | A3, A4, A3→ A4 | 0.22839, 0.1745, 0.61559 |
| 6 | A4, A3,A4 → A2 | 0.96888, 0.31326, 0.61191 |
| 7 | A4, A2 → A3 | 0.061413, 0.97133 |
| 8 | A2, A3→ A3 | 0.013474, 0.98158 |
| 9 | A2, A3, A3 → A4 | 0.18357, 0.58789, 0.26039 |
| 10 | A2, A3, A3 ,A4 → A4 | 0.230846, 0.042254, 0.070553 |
| 11 | A3, A4, A4 → A3 | 0.09823, 0.00237, 0.10643 |
| 12 | A4, A4, A3 → A3 | 0.09832, 0.36008, 0.61727 |
| 13 | A4, A3, A3 → A3 | 0.224562, 0.004632, 0.0084507 |
| 14 | A3, A3, A3 → A2 | 0.02245, 0.31325, 0.97183 |
| 15 | A3, A2 → A1 | 0.49387, 0.44118 |
| 16 | A2, A1 → A1 | 0.02321, 0.61559 |
| 17 | A1, A1→ A4 | 0.11145, 0.042264 |
| 18 | A1, A4 → A6 | 0.03267, 0.00256 |
| 19 | A4, A6 → A7 | 0.01897, 0.027108 |
| 20 | A6, A7 → A4 | 0.57553, 0.39372 |
| 21 | A6, A7, A4 → A7 | 0.37410, 0.09821, 0.67198 |
| 22 | A6, A7, A4, A7 → A7 | 0.1745, 0.061413, 0.97133, 0.49387 |
| 23 | A4, A7 ,A7 → A7 | 0.29990, 0.004599, 0.46357 |
| 24 | A7, A7, A7 → A4 | 0.082003, 0.88125, 0.92345 |
| 25 | A7, A7, A4 → A7 | 0.046111, 0.17091, 0.84638 |
| 26 | A7, A7, A4, A7 → A5 | 0.059782, 0.033079, 0.15022, 0.04442 |
| 27 | A7, A5 → A4 | 0.9760, 0.44421 |
| 28 | A5, A4 → A4 | 0.00236, 0.72221 |
| 29 | A5, A4, A4 → A5 | 0.99733, 0.00228, 0.92540 |
| 30 | A4, A5 → A6 | 0.23716, 0.77197 |
| 31 | A5, A6 → A3 | 0.65246, 0.30856 |
| 32 | A6, A3→ A5 | 0.55816, 0.46357 |

## 3.4 Comparison of Results Obtained with Existing Techniques

The performance of the developed model is evaluated using the performance measures of RMSE and MAPE as shown in equations (2.8) and (2.9) respectively.

## CHAPTER FOUR RESULTS AND DISCUSSIONS

## Introduction

This chapter presents the obtained results for the developed Fuzzy Time Series (FTS) model. Also presented are the results and performances of the benchmark FTS approach of Song and Chissom (1993) and Fuzzy C-Means (FCM) based FTS approach of Yusuf *et al.* (2015), on five data sets.

## Forecasting Results for Car Road Accident

The results are obtained by utilizing the optimal weights, shown in Table 3.6. Table 4.1 shows the result of forecasts obtained for car road accident in Belgium.

Table 4.1: Obtained Model Forecasts Results for Car Road Accident Data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year** | **Actual** | **Forecasted** | **Year** | **Actual** | **Forecasted** |
|  | **Value** | **Value** |  | **Value** | **Value** |
| 1974 | 1574 | - | 1990 | 1574 | 1580 |
| 1975 | 1460 | - | 1991 | 1471 | 1462 |
| 1976 | 1536 | 1538 | 1992 | 1380 | 1382 |
| 1977 | 1597 | 1608 | 1993 | 1346 | 1338 |
| 1978 | 1644 | 1646 | 1994 | 1415 | 1417 |
| 1979 | 1572 | 1560 | 1995 | 1228 | 1229 |
| 1980 | 1616 | 1607 | 1996 | 1122 | 1123 |
| 1981 | 1564 | 1572 | 1997 | 1150 | 1148 |
| 1982 | 1464 | 1463 | 1998 | 1224 | 1223 |
| 1983 | 1479 | 1487 | 1999 | 1173 | 1177 |
| 1984 | 1369 | 1371 | 2000 | 1253 | 1252 |
| 1985 | 1308 | 1315 | 2001 | 1288 | 1288 |
| 1986 | 1456 | 1447 | 2002 | 1145 | 1152 |
| 1987 | 1390 | 1390 | 2003 | 1035 | 1041 |
| 1988 | 1432 | 1434 | 2004 | 953 | 945 |
| 1989 | 1488 | 1484 |  |  |  |

As seen from Table 4.1 there was no forecast for the first two years (1974and1975). This is because at least two preceding historical data are required to forecast any future observation. Also the Fuzzy Set Groups (FSG) associated with the third and fourth future observations were extended to the third and fourth order, respectively to remove ambiguity. The plot for the forecasted training data set and actual training data set is shown in Figure 4.1.

1700

Actual Value

1600

Forecasted Value

1500

1400

1300

1200

1100

1000

900

1976

1982

1988

1994

2000

2006

**Years**

**Number of deaths**

4.1: Plot of Actual and Forecasted Belgium Car Road Accident Data Set

As seen from Figure 4.1, the actual and forecasted plots have very similar trends all through. This implies that the developed FTS model can predict the trends in the given observations.

Table 4.2 shows the results of evaluating on the developed Fuzzy Time Series Model of students enrolment data of Alabama University..

Table 4.2: Obtained Model Forecasts for Student Enrolment Data of Alabama University

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Actual** | **Forecasted** |  | **Actual** | **Forecasted** |
| **Year** | **Value** | **Value** | **Year** | **Value** | **Value** |
| 1971 | 13055 | - | 1982 | 15433 | 15300 |
| 1972 | 13563 | - | 1983 | 15497 | 15176 |
| 1973 | 13867 | 14701 | 1984 | 15145 | 15290 |
| 1974 | 14696 | 14701 | 1985 | 15163 | 15091 |
| 1975 | 15460 | 15247 | 1986 | 15984 | 16056 |
| 1976 | 15311 | 15204 | 1987 | 16859 | 17045 |
| 1977 | 15603 | 15966 | 1988 | 18150 | 18521 |
| 1978 | 15861 | 15484 | 1989 | 18970 | 19403 |
| 1979 | 16807 | 16841 | 1990 | 19328 | 19018 |
| 1980 | 16919 | 17031 | 1991 | 19337 | 19117 |
| 1981 | 16388 | 16528 | 1992 | 18876 | 19102 |

Similarly, Table 4.2 has no forecast for two years (1971and1972); due to the fact that at least two historical data are required to forecast any future observation. To further verify the developed FTS model, a plot for the forecasted data set and actual data set for Alabama University Student Enrolment is shown in Figure 4.2.

**Number of Enrolment**

Figure 4.2: Plot of Actual and Forecasted Alabama University Student Enrolment Data Set

20000

Actual Enrolment

19000

Forecasted Enrolment

18000

17000

16000

15000

14000

1973

1978

1983

**Years**

1988

1993

As seen from Figure 4.2, the forecasted plot and the actual observations plot follow a similar trend; except for mismatch in the years 1977, 1983 and 1990. The points of mismatch on the plots of actual enrolment and forecasted enrolment could be as a result of the parameters set for the CSO-C algorithm or as a result of outliers in the data set. Table 4-3 shows the results obtained when TAIFEX data is applied on the Developed Fuzzy Time Series (FTS) model.

Table 4.3: Developed Model Forecasts for TAIFEX Data

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Date** | **Actual Values** | **Forecasted** |  | **Date** | **Actual** | **Forecasted** |
| **Values** | **Values** | **Values** |
| 1 | 11.09.1998 | 6726.50 | - | 9 | 22.09.1998 | 6926.00 | 6969.37 |
| 2 | 12.09.1998 | 6774.55 | - | 10 | 23.09.1998 | 6852.00 | 6837.16 |
| 3 | 15.09.1998 | 6762.00 | 6727.52 | 11 | 24.09.1998 | 6890.00 | 6844.69 |
| 4 | 16.09.1998 | 6952.75 | 6891.48 | 12 | 25.09.1998 | 6871.00 | 6909.2 |
| 5 | 17.09.1998 | 6906.00 | 6963.83 | 13 | 28.09.1998 | 6840.00 | 6810.65 |
| 6 | 18.09.1998 | 6842.00 | 6820.84 | 14 | 29.09.1998 | 6806.00 | 6809.38 |
| 7 | 19.09.1998 | 7039.00 | 7091.37 | 15 | 30.09.1998 | 6787.00 | 6823.89 |
| 8 | 21.09.1998 | 6861.00 | 6927.67 |  |  |  |  |

Similarly, the first two indices have no forecast due to the necessary extension for a more accurate forecast. The plot for the forecasted data set and actual data set for TAIFEX is shown in Figure 4.3.



7150

7100

7050

Actual TAIFEX

Forecasted TAIFEX

7000

6950

6900

6850

6800

6750

6700

6650

1

3

5

7

**Index**

9

11

13

**TAIFEX Values**

Figure 4.3: Plot of Actual and Forecasted Values for TAIFEX Data Set

Similarly, the actual and the forecasted plots are very similar; except for indices 9, 10, 11, and 12.Table 4.4 shows the forecast obtained when the developed fuzzy time series model is applied on UNIMAID student enrolment data set.

Table 4.4: Developed Model Forecasting UNIMAID Student Enrolment Data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year** | **Actual**  **Data** | **Forecasted**  **Data** | **Year** | **Actual**  **Data** | **Forecasted**  **Data** |
| 1976 | 743 | - | 1985 | 5800 | 5803 |
| 1977 | 1128 | - | 1986 | 6168 | 6163 |
| 1978 | 1882 | 1883 | 1987 | 6711 | 6697 |
| 1979 | 2500 | 2789 | 1988 | 7238 | 7254 |
| 1980 | 2925 | 2918 | 1989 | 7687 | 7651 |
| 1981 | 3251 | 3274 | 1990 | 7960 | 8022 |
| 1982 | 4561 | 4523 | 1991 | 8302 | 8273 |
| 1983 | 5329 | 5351 | 1992 | 9884 | 9933 |
| 1984 | 5719 | 5722 | 1993 | 11410 | 11334 |

Table 4.4 also has no forecast for the years (1976-1977). The plot for the forecasted data set and actual data set for UNIMAID Student Enrolment is shown in Figure 4.4.

**Number of Enrolment**

Figure 4.4: Plot of Actual and Forecasted UNIMAID Student Enrolment Data Set



12000

10000

Actual

Forecasted

8000

6000

4000

2000

0

1975

1980

1985

**Years**

1990

1995

As seen in Figure 4.4, mismatch only occurred at years 1978 and 1979. But the forecasted and actual plots have very similar patterns.

Table 4.5: Developed Model Forecasts for Jigawa Monthly Temperature Data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year** | **Historical**  **Data** | **Forecasted**  **Data** | **Year** | **Historical**  **Data** | **Forecasted**  **Data** |
| 1982 | 30.7 | - | 1998 | 32.0 | 32.2 |
| 1983 | 31.5 | - | 1999 | 33.0 | 32.4 |
| 1984 | 32.8 | 33.0 | 2000 | 34.9 | 35.2 |
| 1985 | 28.6 | 28.4 | 2001 | 32.3 | 32.7 |
| 1986 | 29.0 | 28.6 | 2002 | 34.2 | 34.5 |
| 1987 | 34.2 | 34.3 | 2003 | 34.1 | 33.8 |
| 1988 | 28.0 | 27.5 | 2004 | 35.9 | 35.5 |
| 1989 | 29.3 | 29.8 | 2005 | 32.4 | 32.4 |
| 1990 | 29.1 | 29.6 | 2006 | 35.8 | 35.4 |
| 1991 | 29.8 | 29.9 | 2007 | 31.9 | 32.0 |
| 1992 | 28.9 | 28.4 | 2008 | 30.9 | 31.4 |
| 1993 | 30.7 | 30.4 | 2009 | 33.2 | 33.8 |
| 1994 | 28.3 | 28.5 | 2010 | 29.5 | 29.3 |
| 1995 | 28.9 | 28.8 | 2011 | 34.9 | 34.8 |
| 1996 | 29.0 | 29.2 | 2012 | 29.4 | 29.8 |
| 1997 | 30.0 | 29.9 | 2013 | 32.6 | 32.0 |

Table 4.5 shows the results obtained when the Developed model was applied to monthly temperature data of Jigawa state.

Similarly, the plot of the actual and forecasted results is shown in Figure 4.5

Actual Temperature

Forecasted Temperature

36

34

32

30

28

26

1985

1990

1995

2000

**Years**

2005

2010

2015

**Temperature (⁰C)**

Figure 4.5: Plot of Actual and Forecasted Jigawa Monthly Temperature Data Set

As seen in Figure 4.5, the plots of the actual and forecasted observations showed very similar pattern.

## Validation

In order to verify the forecasting accuracy of the Developed Fuzzy Time Series (FTS) model, its performance measures of Root Mean Square Error (RMSE) and Mean Absolute Percentage Error

(MAPE) are determined. Subsequently, these measures for the Developed model are compared with those of some existing models.

Table 4.6: Calculation of RMSE and MAPE of Forecast for Belgium Car Road Accident Data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **S/No.** | **Date** | **Actual**  **Data** | **Forecasted**  **Data** | **SEt** | **AEt (\*100%)** |
| 1 | 1974 | 1574 | - | - | - |
| 2 | 1975 | 1460 | - | - | - |
| 3 | 1976 | 1536 | 1542 | 36 | 0.003915 |
| 4 | 1977 | 1597 | 1588 | 81 | 0.005636 |
| 5 | 1978 | 1644 | 1650 | 36 | 0.003650 |
| 6 | 1979 | 1572 | 1560 | 144 | 0.007634 |
| 7 | 1980 | 1616 | 1607 | 81 | 0.005569 |
| 8 | 1981 | 1564 | 1572 | 64 | 0.005115 |
| 9 | 1982 | 1464 | 1463 | 1 | 0.000683 |
| 10 | 1983 | 1479 | 1487 | 64 | 0.005409 |
| 11 | 1984 | 1369 | 1371 | 4 | 0.001461 |
| 12 | 1985 | 1308 | 1315 | 49 | 0.005352 |
| 13 | 1986 | 1456 | 1447 | 81 | 0.006181 |
| 14 | 1987 | 1390 | 1390 | 0 | 0.00000 |
| 15 | 1988 | 1432 | 1434 | 4 | 0.001397 |
| 16 | 1989 | 1488 | 1484 | 16 | 0.002688 |
| 17 | 1990 | 1574 | 1580 | 36 | 0.003812 |
| 18 | 1991 | 1471 | 1462 | 81 | 0.006118 |
| 19 | 1992 | 1380 | 1382 | 4 | 0.001449 |
| 20 | 1993 | 1346 | 1338 | 64 | 0.005944 |
| 21 | 1994 | 1415 | 1417 | 4 | 0.001413 |
| 22 | 1995 | 1228 | 1229 | 1 | 0.000814 |
| 23 | 1996 | 1122 | 1123 | 1 | 0.000891 |
| 24 | 1997 | 1150 | 1148 | 4 | 0.001739 |
| 25 | 1998 | 1224 | 1223 | 1 | 0.000817 |
| 26 | 1999 | 1173 | 1177 | 16 | 0.003410 |
| 27 | 2000 | 1253 | 1252 | 1 | 0.000798 |
| 28 | 2001 | 1288 | 1288 | 0 | 0.00000 |
| 29 | 2002 | 1145 | 1152 | 49 | 0.006114 |
| 30 | 2003 | 1035 | 1041 | 36 | 0.005797 |
| 31 | 2004 | 953 | 945 | 64 | 0.008395 |
|  |  |  | RMSE=5.931 | | MAPE=.34607% |

These existing models include; Chen’s (1996) model and Yusuf *et al* (2015). Chen’s (1996) model and Yusuf et al (2015) are training models.

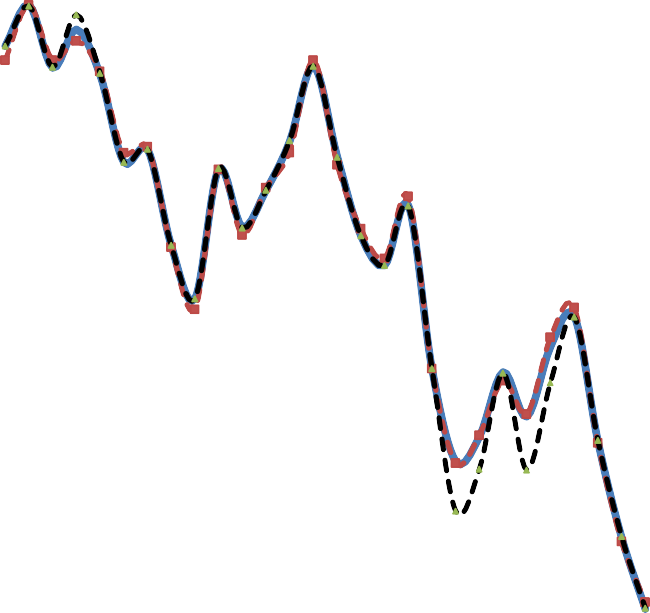
To determine the Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) of these models, equations (2.8) and (2.9) respectively utilized. Table 4-6 shows the calculation of the performance measures for the Developed fuzzy time series model.

AEt is the Absolute Error and SEt is the Squared Error. The result shows that statistical measures of RMSE=5.931 and MAPE=0.34607% were obtained using the Developed model. Table 4.7 shows the comparison between the performance of the Developed model and that of some previously developed FTS models in forecasting car road accident in Belgium.

Table 4.7: Comparison between the Developed Model and that of Previous Works for Accident Data

|  |  |  |
| --- | --- | --- |
|  | **Performance Index** | |
| **Methods** |
|  | RMSE | MAPE (%) |
| Jilani *et al* (2007) | 83.12 | 5.06 |
| Egrioglu *et al* (2010) | 85.35 | 5.25 |
| Uslu *et al* (2014) | 41.61 | 2.29 |
| Yusuf *et al* (2015) | 19.2 | 0.67 |
| Developed FTS model | 5.93 | 0.34 |

As seen in Table 4.7, the developed FTS model outperforms other previous models in terms of accuracy. Figure 4.6 shows the comparison plot between the Developed FTS forecasts and that of Yusuf *et al* (2015).



1700

1600

1500

1400

1300

1200

Actual

Proposed Model Yusuf et. al (2015)

1100

1000

900

1975

1980

1985

1990

1995

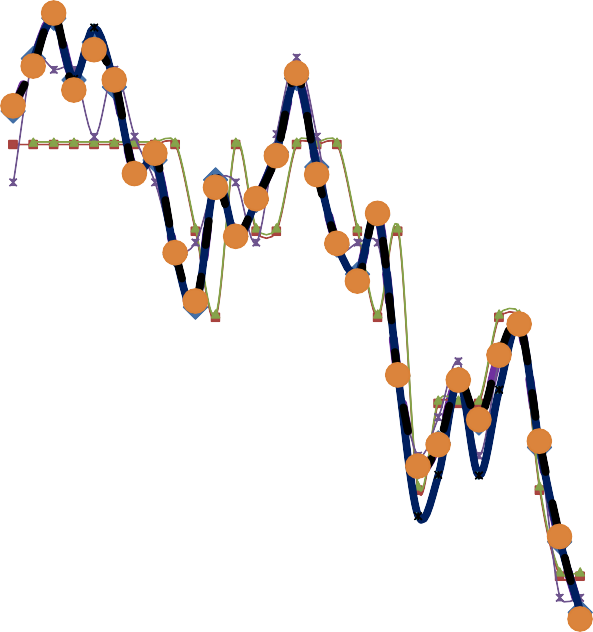
2000

2005

2010

Figure 4.6: Plot of Actual and Forecasted values for Car Road Accident

As seen in Figure 4.6 the proposed (developed) models curve in grey dash curve followed the rise and fall pattern of the actual curve in black thick continuous curve.



1700

1600

1500

1400

Actual Data

Jilani et al

1300

1200

Egrioglu et al

Uslu et al Yusuf et al

Developed FTS model

1100

1000

900

1970 1975 1980 1985 1990 1995 2000 2005 2010

Figure 4.7: Comparison between the Developed Model and that of Previous Works for Car Road Accident

Figure 4.7 shows the actual curve in continuous black curve and the developed models curve in black dotted lines following the pattern of the actual curve. Table 4.8 shows the calculation of the performance measures for the Developed fuzzy time series model when used to forecast for the enrolment in University of Alabama.

Table 4.8: Calculation of RMSE and MAPE of Forecast for Alabama University Student Enrolment Data

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Date** | **Actual Data** | **Forecasted Data** | **SEt** | **AEt** |
| 1971 | 13055 | - | - | - |
| 1972 | 13563 | - | - | - |
| 1973 | 13867 | 13874 | 49 | 0.0005048 |
| 1974 | 14696 | 14701 | 25 | 0.0003402 |
| 1975 | 15460 | 15453 | 49 | 0.00045278 |
| 1976 | 15311 | 15307 | 16 | 0.00026125 |
| 1977 | 15603 | 15611 | 64 | 0.00051272 |
| 1978 | 15861 | 15860 | 1 | 0.00006305 |
| 1979 | 16807 | 16809 | 4 | 0.000119 |
| 1980 | 16919 | 16921 | 4 | 0.00011821 |
| 1981 | 16388 | 16393 | 25 | 0.0003051 |
| 1982 | 15433 | 15430 | 9 | 0.00019439 |
| 1983 | 15497 | 15493 | 16 | 0.00025811 |
| 1984 | 15145 | 15150 | 25 | 0.00033014 |
| 1985 | 15163 | 15152 | 121 | 0.00072545 |
| 1986 | 15984 | 15985 | 1 | 0.00006256 |
| 1987 | 16859 | 16858 | 1 | 0.00005932 |
| 1988 | 18150 | 18162 | 144 | 0.00066116 |
| 1989 | 18970 | 18961 | 81 | 0.00047443 |
| 1990 | 19328 | 19340 | 144 | 0.00062086 |
| 1991 | 19337 | 19349 | 144 | 0.00062057 |
| 1992 | 18876 | 18882 | 36 | 0.00031786 |
|  |  | RMSE = 6.669 | | MAPE = 0.033% |

The result shows that statistical measures of RMSE=6.669 and MAPE=0.033 % were obtained using the Developed model. Table 4.9 shows the comparative result between the Developed FTS model and previous models.

Table 4.9: Comparison between the Developed Model and that of Previous Works for Alabama University Student Enrolment Data

|  |  |  |
| --- | --- | --- |
|  | **Performance Index** | |
| **Methods** |
|  | RMSE | MAPE (%) |
| Song & Chissom (1993) | 650 | 3.22 |
| Chen (1996) | 638 | 3.11 |
| Haurng *et al* (2006) | 478 | 2.20 |
| Huarng (2001) | 476 | 2.45 |
| Uslu *et al* (2014) | 178 | 0.90 |
| Yusuf *et al* (2015) | 7.02 | 0.04 |
| Developed FTS model | 5.06 | 0.046 |

As seen in Table 4.9, the developed model outperformed other previous FTS models except for Yusuf *et al.* (2015); in terms of MAPE. This implies that Yusuf et al (2015) is a better training model than the developed FTS model; for the students’ enrolment in University of Alabama. This can be clearly seen in Figure 4.8.

20000

19000

Actual Enrolment

18000

Proposed Model

Yusuf et al (2015)

17000

16000

15000

14000

1970

1975

1980

1985

1990

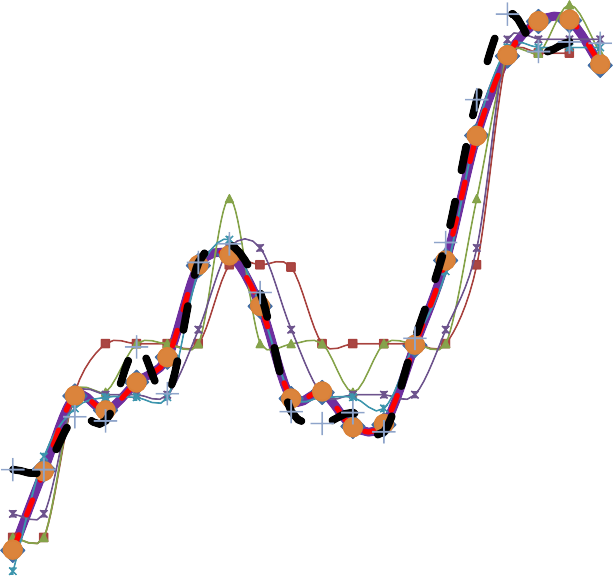
1995

**YEAR**

**No. of Students Enrolled**

Figure 4.8: Comparison of Models for Alabama Students Enrolment

Figure 4.8 shows the plot of the developed (proposed) model in comparison with that of Yusuf *et al* (2015).



20000

19000

18000

17000

16000

15000

Actual Data

Song & Chissom Huarang Huarang et al Uslu et al

Yusuf et al

Developed Model

14000

13000

1970

1975

1980

1985

1990

1995

Figure 4.9: Comparison between the Developed Model and that of Previous Works for Alabama University Student Enrolment Data

As seen in figure 4.9 the actual data curve is in purple color and the developed model curve is in a black dotted curve, While Yusuf *et al* is in red dotted curve.

Table 4.10 shows the calculation of the performance measures for the Developed FTS model when utilized to forecast TAIFEX data.

Table 4.10: Calculation of RMSE and MAPE of Forecast for TAIFEX Data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **S/No.** | **Date** | **Actual Data** | **Forecasted Data** | **SEt** | **AEt** |
| 1 | 04.08.1998 | 6726.5 | - | - | - |
| 2 | 05.08.1998 | 6774.55 | - | - | - |
| 3 | 06.08.1998 | 6762 | 6727.52 | 1188.8704 | 0.005099 |
| 4 | 07.08.1998 | 6952.75 | 6891.48 | 3754.0129 | 0.008812 |
| 5 | 10.08.1998 | 6906 | 6963.83 | 3344.3089 | 0.008374 |
| 6 | 11.08.1998 | 6842 | 6820.84 | 447.7456 | 0.003093 |
| 7 | 12.08.1998 | 7039 | 7091.37 | 2742.6169 | 0.00744 |
| 8 | 13.08.1998 | 6861 | 6927.67 | 4444.8889 | 0.009717 |
| 9 | 14.08.1998 | 6926 | 6969.37 | 1880.9569 | 0.006262 |
| 10 | 15.08.1998 | 6852 | 6837.16 | 220.2256 | 0.002166 |
| 11 | 17.08.1998 | 6890 | 6844.69 | 2052.9961 | 0.006576 |
| 12 | 18.08.1998 | 6871 | 6909.2 | 1459.24 | 0.00556 |
| 13 | 19.08.1998 | 6840 | 6810.65 | 861.4225 | 0.004291 |
| 14 | 20.08.1998 | 6806 | 6809.38 | 11.4244 | 0.000497 |
| 15 | 21.08.1998 | 6787 | 6823.89 | 1360.8721 | 0.005435 |
|  |  |  |  | RMSE=43.868 | MAPE=0.56603% |

Similarly, the result shows that statistical measure of RMSE=43.868 and MAPE=0.56603 % were obtained using the Developed model.

Table 4.11 shows the calculation of the performance measures for the Developed FTS model when utilized to forecast UNIMAID student enrolment.

Table 4.11: Calculation of RMSE and MAPE of Forecast for UNIMAID Student Enrolment Data

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Date** | **Actual Data** | **Forecasted Data** | **SEt** | **AEt** |
| 1976 | 743 | - | - | - |
| 1977 | 1128 | - | - | - |
| 1978 | 1882 | 1883 | 1 | 0.00053135 |
| 1979 | 2500 | 2500 | 0 | 0 |
| 1980 | 2925 | 2924 | 1 | 0.00034188 |
| 1981 | 3251 | 3252 | 1 | 0.0003076 |
| 1982 | 4561 | 4559 | 4 | 0.0004385 |
| 1983 | 5329 | 5332 | 9 | 0.00056296 |
| 1984 | 5719 | 5720 | 1 | 0.00017486 |
| 1985 | 5800 | 5804 | 16 | 0.00068966 |
| 1986 | 6168 | 6167 | 1 | 0.00016213 |
| 1987 | 6711 | 6708 | 9 | 0.00044703 |
| 1988 | 7238 | 7242 | 16 | 0.00055264 |
| 1989 | 7687 | 7686 | 1 | 0.00013009 |
| 1990 | 7960 | 7964 | 16 | 0.00050251 |
| 1991 | 8302 | 8298 | 16 | 0.00048181 |
| 1992 | 9884 | 9885 | 1 | 0.00010117 |
| 1993 | 11410 | 11405 | 25 | 0.00043821 |
|  |  | RMSE = 2.571 | | MAPE = 0.03749% |

As seen in Table 4.11, the result shows a RMSE of 2.571 and MAPE of 0.03749%.

Table 4.12 shows the calculation of the statistical measures for the Developed FTS model when applied on monthly temperature in Jigawa state.

Table 4.12: Calculation of RMSE and MAPE of Forecast for Jigawa Monthly TemperatureData

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Actual**  **Data** | **Forecasted**  **Data** | **SEt** | **AEt** |
| 1982 | 30.7 | - | - | - |
| 1983 | 31.5 | - | - | - |
| 1984 | 32.8 | 33.0 | 0.04 | 0.006098 |
| 1985 | 28.6 | 28.4 | 0.04 | 0.006993 |
| 1986 | 29.0 | 28.6 | 0.16 | 0.013793 |
| 1987 | 34.2 | 34.3 | 0.01 | 0.002924 |
| 1988 | 28.0 | 27.5 | 0.25 | 0.017857 |
| 1989 | 29.3 | 29.8 | 0.25 | 0.017065 |
| 1990 | 29.1 | 29.6 | 0.25 | 0.017182 |
| 1991 | 29.8 | 29.9 | 0.01 | 0.003356 |
| 1992 | 28.9 | 28.4 | 0.25 | 0.017301 |
| 1993 | 30.7 | 30.4 | 0.09 | 0.009772 |
| 1994 | 28.3 | 28.5 | 0.04 | 0.007067 |
| 1995 | 28.9 | 28.8 | 0.01 | 0.00346 |
| 1996 | 29.0 | 29.2 | 0.04 | 0.006897 |
| 1997 | 30.0 | 29.9 | 0.01 | 0.003333 |
| 1998 | 32.0 | 32.2 | 0.04 | 0.00625 |
| 1999 | 33.0 | 32.4 | 0.36 | 0.018182 |
| 2000 | 34.9 | 35.2 | 0.09 | 0.008596 |
| 2001 | 32.3 | 32.7 | 0.16 | 0.012384 |
| 2002 | 34.2 | 34.5 | 0.09 | 0.008772 |
| 2003 | 34.1 | 33.8 | 0.09 | 0.008798 |
| 2004 | 35.9 | 35.5 | 0.16 | 0.011142 |
| 2005 | 32.4 | 32.4 | 0.00 | 0.000000 |
| 2006 | 35.8 | 35.4 | 0.16 | 0.011173 |
| 2007 | 31.9 | 32.0 | 0.01 | 0.003135 |
| 2008 | 30.9 | 31.4 | 0.25 | 0.016181 |
| 2009 | 33.2 | 33.8 | 0.36 | 0.018072 |
| 2010 | 29.5 | 29.3 | 0.04 | 0.00678 |
| 2011 | 34.9 | 34.8 | 0.01 | 0.002865 |
| 2012 | 29.4 | 29.8 | 0.16 | 0.013605 |
| 2013 | 32.6 | 32.0 | 0.36 | 0.018405 |
|  |  | RMSE=0.357 | | MAPE=0.99945% |

As seen in Table 4.12, statistical measures of RMSE=0.357 and MAPE = 0.99945% were obtained.

## Significance of Forecasting Results

With regard to the Developed model, using the two statistical performance measures of Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) and its comparison with Yusuf *et al* (2015) (RMSE=19.2, MAPE=0.67%) while the Developed model achieved a higher accuracy of (RMSE=5.931, MAPE=0.34%) in car road accident prediction. While,in the prediction of Alabama student enrolment Yusuf *et al* (2015) (RMSE=7.02, MAPE=0.04%) the improvement in the Developed model’s prediction in this regard is (RMSE=6.66, MAPE=0.033%). This is due to the fact that, the Developed hybrid model has the lowest square loss and absolute loss. In terms of mismatch this can easily be noticed by a visual inspection of the drawn plots.

## CHAPTER FIVE

**SUMMARY, CONCLUSION AND RECOMMENDATIONS**

## Summary

In this work, a hybrid model that integrates Cat Swarm Optimization-Clustering (CSO-C) and Particle Swarm Optimization (PSO) into Fuzzy Time Series (FTS) forecasting model to improve forecasting accuracy was developed. The developed model was then applied to forecast five different data sets of which the last two among the five data sets are: University of Maiduguri (UNIMAID) student enrolment data and Jigawa State monthly temperature data. Cat Swarm Optimization Clustering (CSO-C) algorithm was coded in MATLAB and applied to generate seven unequal partitions (cluster centres) for the five data sets obtained. Based on the unequal partitions and membership degrees (partition matrix) obtained, a total of one hundred and four fuzzy “if”- rules were generated. By utilizing PSO coded in MATLAB, weights of forecasting rules were tuned to match the future data they represent. Finally, the forecasts were obtained and forecasting performances of the proposed model was compared with that of Chen’s (1996) and Yusuf *et al* (2015) models.

## Significant Contributions

* + 1. Development of an FTS model that incorporates Cat Swarm Optimization Clustering (CSO-C) and Particle Swarm Optimization (PSO) which speeds up convergence time in comparison with other existing clustering-based FTS model.
    2. Development of a hybrid fuzzy time series forecasting model that has improved forecasting performance in terms of Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE). In comparison with the bench mark FTS model by Song and Chissom in 1993, Chen’s FTS model in 1996 and the work of Yusuf *et al* 2015.

## Conclusion

Researchers’ observation has revealed that, objectively partitioning universe of discourse and the use of optimization technique to improve the defuzzification process affects forecasting accuracy. The developed model considered eliminating the need to define universe of discourse, learning memberships in hidden data structures, portioning the universe of discourse objectively and optimizing the defuzzification process. Cat Swarm Optimization Clustering (CSO-C) algorithm was developed to objectively partition the universe of discourse and learn memberships in five data sets namely: Belgium car road accident data obtained from the work of Yusuf *et al* (2015), Alabama University enrolment data obtained from the work of Song and Chissom (1993), TAIFEX data obtained from the work of Bas *et al* (2015), UNIMAID enrolment data obtained from academic planning unit of University of Maiduguri and Jigawa state monthly temperature data obtained from Nigerian Meteorological Agency (NiMet). To obtain unique fuzzy relations, Fuzzy Set Groups (FSGs) were generated for the five data sets and converted to ‘if-then’ forecasting rule. Then, particle swarm optimization algorithm was developed using MATLAB. By comparing the proposed hybrid model’s result with benchmark work and that of Yusuf *et al* (2015) forecasting models, using RMSE and MAPE criteria, it was observed that the proposed forecasting model provides more accurate forecasts.

## Recommendations for Further Work

Further work should consider the following for possible improvement:

* + 1. More comparisons should be made with other existing models to further validate the proposed hybrid model.
    2. Other fuzzy clustering techniques should be applied and compared with Cat Swarm Optimization Clustering (CSO-C) to determine the most suitable clustering technique for univariate data.

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## APPENDICES

**Appendix A: Belgium Car Road Accident Data**

|  |  |  |  |
| --- | --- | --- | --- |
| **Years** | **Actual** | **Years** | **Actual** |
| 1974  1975  1976  1977  1978  1979  1980  1981  1982  1983  1984  1985  1986  1987  1988  1989 | 1574  1460  1536  1597  1644  1572  1616  1564  1464  1479  1369  1308  1456  1390  1432  1488 | 1990  1991  1992  1993  1994  1995  1996  1997  1998  1999  2000  2001  2002  2003  2004 | 1574  1471  1380  1346  1415  1228  1122  1150  1224  1173  1253  1288  1145  1035  953 |

## Appendix B: Benchmark Data of Enrolment of University of Alabama

|  |  |  |  |
| --- | --- | --- | --- |
| **Years** | **Historical**  **Data** | **Years** | **Historical**  **Data** |
| 1971  1972  1973  1974  1975  1976  1977  1978  1979  1980  1981 | 13055  13563  13867  14696  15460  15311  15603  15861  16807  16919  16388 | 1982  1983  1984  1985  1986  1987  1988  1989  1990  1991  1992 | 15433  15497  15245  15163  15984  16859  18150  18970  19328  19337  18872 |

**Appendix C: Taiwan Future Exchange (TAIFEX) Data**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Date | TAIFEX | Date | TAIFEX | Date | TAIFEX |
| 03.08.1998 | 7552.00 | 24.08.1998 | 6955.00 | 11.09.1998 | 6726.50 |
| 04.08.1998 | 7560.00 | 25.08.1998 | 6949.00 | 12.09.1998 | 6774.55 |
| 05.08.1998 | 7487.00 | 26.08.1998 | 6790.00 | 15.09.1998 | 6762.00 |
| 06.08.1998 | 7462.00 | 27.08.1998 | 6835.00 | 16.09.1998 | 6952.75 |
| 07.08.1998 | 7515.00 | 28.08.1998 | 6695.00 | 17.09.1998 | 6906.00 |
| 10.08.1998 | 7365.00 | 29.08.1998 | 6728.00 | 18.09.1998 | 6842.00 |
| 11.08.1998 | 7360.00 | 31.08.1998 | 6566.00 | 19.09.1998 | 7039.00 |
| 12.08.1998 | 7330.00 | 01.09.1998 | 6409.00 | 21.09.1998 | 6861.00 |
| 13.08.1998 | 7291.00 | 02.09.1998 | 6430.00 | 22.09.1998 | 6926.00 |
| 14.08.1998 | 7320.00 | 03.09.1998 | 6200.00 | 23.09.1998 | 6852.00 |
| 15.08.1998 | 7320.00 | 04.09.1998 | 6403.20 | 24.09.1998 | 6890.00 |
| 17.08.1998 | 7219.00 | 05.09.1998 | 6697.50 | 25.09.1998 | 6871.00 |
| 18.08.1998 | 7220.00 | 07.09.1998 | 6722.30 | 28.09.1998 | 6840.00 |
| 19.08.1998 | 7285.00 | 08.09.1998 | 6859.40 | 29.09.1998 | 6806.00 |
| 20.08.1998 | 7274.00 | 09.09.1998 | 6769.60 | 30.09.1998 | 6787.00 |
| 21.08.1998 | 7225.00 | 10.09.1998 | 6709.75 |  |  |

## Appendix D: University of Maiduguri Student Enrolment Data

|  |  |  |  |
| --- | --- | --- | --- |
| **Year** | **Actual Data** | **Year** | **Actual Data** |
| 1976 | 743 | 1985 | 5800 |
| 1977 | 1128 | 1986 | 6168 |
| 1978 | 1882 | 1987 | 6711 |
| 1979 | 2500 | 1988 | 7238 |
| 1980 | 2925 | 1989 | 7687 |
| 1981 | 3251 | 1990 | 7960 |
| 1982 | 4561 | 1991 | 8302 |
| 1983 | 5329 | 1992 | 9884 |
| 1984 | 5719 | 1993 | 11410 |

**Appendix E: Jigawa State Monthly Temperature Data**

|  |  |  |  |
| --- | --- | --- | --- |
| **Date** | **Training**  **Data Set** | **Date** | **Training Data Set** |
| 1982 | 36.2 | 1998 | 36.9 |
| 1983 | 34.8 | 1999 | 37.8 |
| 1984 | 36.9 | 2000 | 38.4 |
| 1985 | 35.6 | 2001 | 36.7 |
| 1986 | 36.4 | 2002 | 38.9 |
| 1987 | 33.7 | 2003 | 38.1 |
| 1988 | 35.0 | 2004 | 38.0 |
| 1989 | 35.3 | 2005 | 36.7 |
| 1990 | 36.3 | 2006 | 39.5 |
| 1991 | 37.0 | 2007 | 37.3 |
| 1992 | 35.5 | 2008 | 36.6 |
| 1993 | 35.2 | 2009 | 36.9 |
| 1994 | 35.8 | 2010 | 37.4 |
| 1995 | 34.5 | 2011 | 37.7 |
| 1996 | 32.4 | 2012 | 36.1 |
| 1997 | 33.2 | 2013 | 37.5 |

## Appendix F: Membership Values for UNIMAID Data

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | **K1** | **K2** | **K3** | **K4** | **K5** | **K6** | **K7** |
| 1976 | 1.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1977 | 0.83 | 0.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1978 | 0.49 | 0.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1979 | 0.21 | 0.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1980 | 0.02 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1981 | 0.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1982 | 0.00 | 0.26 | 0.44 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1983 | 0.00 | 0.00 | 0.79 | 0.14 | 0.00 | 0.00 | 0.00 |
| 1984 | 0.00 | 0.00 | 0.96 | 0.32 | 0.11 | 0.00 | 0.00 |
| 1985 | 0.00 | 0.00 | 1.00 | 0.35 | 0.15 | 0.00 | 0.00 |
| 1986 | 0.00 | 0.00 | 0.83 | 0.52 | 0.32 | 0.00 | 0.00 |
| 1987 | 0.00 | 0.00 | 0.59 | 0.76 | 0.56 | 0.00 | 0.00 |
| 1988 | 0.00 | 0.00 | 0.35 | 1.00 | 0.80 | 0.00 | 0.00 |
| 1989 | 0.00 | 0.00 | 0.15 | 0.80 | 1.00 | 0.01 | 0.00 |
| 1990 | 0.00 | 0.00 | 0.03 | 0.68 | 0.88 | 0.13 | 0.00 |
| 1991 | 0.00 | 0.00 | 0.00 | 0.52 | 0.72 | 0.29 | 0.00 |
| 1992 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 1.00 | 0.31 |
| 1993 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.31 | 1.00 |

**Appendix G: Membership Values for Jigawa Monthly Temperature Data (July)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | **K1** | **K2** | **K3** | **K4** | **K5** | **K6** | **K7** |
| 1982 | 1.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1983 | 0.83 | 0.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1984 | 0.49 | 0.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1985 | 0.21 | 0.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1986 | 0.02 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1987 | 0.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1988 | 0.00 | 0.26 | 0.44 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1989 | 0.00 | 0.00 | 0.79 | 0.14 | 0.00 | 0.00 | 0.00 |
| 1990 | 0.00 | 0.00 | 0.96 | 0.32 | 0.11 | 0.00 | 0.00 |
| 1991 | 0.00 | 0.00 | 1.00 | 0.35 | 0.15 | 0.00 | 0.00 |
| 1992 | 0.00 | 0.00 | 0.83 | 0.52 | 0.32 | 0.00 | 0.00 |
| 1993 | 0.00 | 0.00 | 0.59 | 0.76 | 0.56 | 0.00 | 0.00 |
| 1994 | 0.00 | 0.00 | 0.35 | 1.00 | 0.80 | 0.00 | 0.00 |
| 1995 | 0.00 | 0.00 | 0.15 | 0.80 | 1.00 | 0.01 | 0.00 |
| 1996 | 0.00 | 0.00 | 0.03 | 0.68 | 0.88 | 0.13 | 0.00 |
| 1997 | 0.00 | 0.00 | 0.00 | 0.52 | 0.72 | 0.29 | 0.00 |
| 1998 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 1.00 | 0.31 |
| 1999 | 0.00 | 0.00 | 0.59 | 0.76 | 0.56 | 0.00 | 0.00 |
| 2000 | 0.00 | 0.00 | 0.35 | 1.00 | 0.80 | 0.00 | 0.00 |
| 2001 | 0.00 | 0.00 | 0.15 | 0.80 | 1.00 | 0.01 | 0.00 |
| 2002 | 0.00 | 0.00 | 0.03 | 0.68 | 0.88 | 0.13 | 0.00 |
| 2003 | 0.00 | 0.00 | 0.00 | 0.52 | 0.72 | 0.29 | 0.00 |
| 2004 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 1.00 | 0.31 |
| 2005 | 0.02 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2006 | 0.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2007 | 0.00 | 0.26 | 0.44 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2008 | 0.00 | 0.00 | 0.79 | 0.14 | 0.00 | 0.00 | 0.00 |
| 2009 | 0.00 | 0.00 | 0.96 | 0.32 | 0.11 | 0.00 | 0.00 |
| 2010 | 0.00 | 0.00 | 1.00 | 0.35 | 0.15 | 0.00 | 0.00 |
| 2011 | 0.00 | 0.00 | 0.83 | 0.52 | 0.32 | 0.00 | 0.00 |
| 2012 | 0.00 | 0.00 | 0.00 | 0.52 | 0.72 | 0.29 | 0.00 |
| 2013 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 1.00 | 0.31 |

## Appendix H: Membership Values for Road Accident Data

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | **k1** | **k2** | **k3** | **k4** | **k5** | **k6** | **k7** |
| 1975 | 0.87 | 0.96 | 0.99 | 0.99 | 0.95 | 0.93 | 0.92 |
| 1976 | 0.84 | 0.93 | 0.95 | 0.97 | 0.98 | 0.96 | 0.95 |
| 1977 | 0.81 | 0.90 | 0.93 | 0.95 | 0.99 | 0.99 | 0.98 |
| 1978 | 0.79 | 0.88 | 0.90 | 0.93 | 0.97 | 0.99 | 1.00 |
| 1979 | 0.82 | 0.91 | 0.94 | 0.96 | 1.00 | 0.98 | 0.97 |
| 1980 | 0.80 | 0.89 | 0.92 | 0.94 | 0.98 | 1.00 | 0.99 |
| 1981 | 0.82 | 0.92 | 0.94 | 0.96 | 1.00 | 0.98 | 0.96 |
| 1982 | 0.87 | 0.96 | 0.99 | 0.99 | 0.95 | 0.93 | 0.92 |
| 1983 | 0.86 | 0.96 | 0.98 | 1.00 | 0.96 | 0.94 | 0.93 |
| 1984 | 0.91 | 1.00 | 0.97 | 0.95 | 0.91 | 0.89 | 0.88 |
| 1985 | 0.94 | 0.97 | 0.94 | 0.92 | 0.88 | 0.86 | 0.85 |
| 1986 | 0.87 | 0.97 | 0.99 | 0.99 | 0.95 | 0.93 | 0.92 |
| 1987 | 0.90 | 1.00 | 0.98 | 0.96 | 0.92 | 0.90 | 0.89 |
| 1988 | 0.88 | 0.98 | 1.00 | 0.98 | 0.94 | 0.92 | 0.90 |
| 1989 | 0.86 | 0.95 | 0.97 | 1.00 | 0.96 | 0.94 | 0.93 |
| 1990 | 0.82 | 0.91 | 0.94 | 0.96 | 1.00 | 0.98 | 0.97 |
| 1991 | 0.87 | 0.96 | 0.98 | 1.00 | 0.95 | 0.93 | 0.92 |
| 1992 | 0.91 | 1.00 | 0.98 | 0.96 | 0.91 | 0.89 | 0.88 |
| 1993 | 0.92 | 0.98 | 0.96 | 0.94 | 0.90 | 0.88 | 0.87 |
| 1994 | 0.89 | 0.98 | 0.99 | 0.97 | 0.93 | 0.91 | 0.90 |
| 1995 | 0.97 | 0.93 | 0.91 | 0.89 | 0.84 | 0.83 | 0.81 |
| 1996 | 0.98 | 0.88 | 0.86 | 0.84 | 0.80 | 0.78 | 0.77 |
| 1997 | 0.99 | 0.90 | 0.87 | 0.85 | 0.81 | 0.79 | 0.78 |
| 1998 | 0.98 | 0.93 | 0.91 | 0.89 | 0.84 | 0.82 | 0.81 |
| 1999 | 1.00 | 0.91 | 0.88 | 0.86 | 0.82 | 0.80 | 0.79 |
| 2000 | 0.96 | 0.94 | 0.92 | 0.90 | 0.86 | 0.84 | 0.82 |
| 2001 | 0.95 | 0.96 | 0.94 | 0.91 | 0.87 | 0.85 | 0.84 |
| 2002 | 0.99 | 0.89 | 0.87 | 0.85 | 0.81 | 0.79 | 0.78 |
| 2003 | 0.94 | 0.84 | 0.82 | 0.80 | 0.76 | 0.74 | 0.73 |
| 2004 | 0.90 | 0.81 | 0.78 | 0.76 | 0.72 | 0.70 | 0.69 |

**Appendix I: Membership Values for Student Enrolment Data**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | **k1** | **k2** | **k3** | **k4** | **k5** | **k6** | **k7** |
| 1971 | 1.00 | 0.77 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1972 | 0.77 | 1.00 | 0.28 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1973 | 0.63 | 0.86 | 0.42 | 0.10 | 0.00 | 0.00 | 0.00 |
| 1974 | 0.26 | 0.49 | 0.79 | 0.48 | 0.00 | 0.00 | 0.00 |
| 1975 | 0.00 | 0.15 | 0.87 | 0.82 | 0.34 | 0.00 | 0.00 |
| 1976 | 0.00 | 0.21 | 0.93 | 0.75 | 0.28 | 0.00 | 0.00 |
| 1977 | 0.00 | 0.08 | 0.80 | 0.88 | 0.41 | 0.00 | 0.00 |
| 1978 | 0.00 | 0.00 | 0.69 | 1.00 | 0.52 | 0.00 | 0.00 |
| 1979 | 0.00 | 0.00 | 0.26 | 0.57 | 0.95 | 0.40 | 0.00 |
| 1980 | 0.00 | 0.00 | 0.21 | 0.52 | 1.00 | 0.45 | 0.00 |
| 1981 | 0.00 | 0.00 | 0.45 | 0.76 | 0.76 | 0.21 | 0.00 |
| 1982 | 0.00 | 0.16 | 0.88 | 0.81 | 0.33 | 0.00 | 0.00 |
| 1983 | 0.00 | 0.13 | 0.85 | 0.84 | 0.36 | 0.00 | 0.00 |
| 1984 | 0.06 | 0.29 | 0.99 | 0.68 | 0.20 | 0.00 | 0.00 |
| 1985 | 0.05 | 0.28 | 1.00 | 0.69 | 0.21 | 0.00 | 0.00 |
| 1986 | 0.00 | 0.00 | 0.63 | 0.95 | 0.58 | 0.03 | 0.00 |
| 1987 | 0.00 | 0.00 | 0.24 | 0.55 | 0.97 | 0.42 | 0.00 |
| 1988 | 0.00 | 0.00 | 0.00 | 0.00 | 0.45 | 1.00 | 0.47 |
| 1989 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.63 | 0.84 |
| 1990 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.47 | 1.00 |
| 1991 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.47 | 1.00 |
| 1992 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.67 | 0.79 |

## Appendix J: Membership Values for TAIFEX Data

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | **k1** | **k2** | **k3** | **k4** | **k5** | **k6** | **k7** |
| 03.08.1998 | 1.00 | 0.96 | 0.93 | 0.92 | 0.90 | 0.89 | 0.85 |
| 04.08.1998 | 0.99 | 0.96 | 0.93 | 0.93 | 0.91 | 0.90 | 0.86 |
| 05.08.1998 | 0.97 | 0.99 | 0.96 | 0.95 | 0.93 | 0.92 | 0.88 |
| 06.08.1998 | 0.98 | 0.98 | 0.95 | 0.94 | 0.93 | 0.91 | 0.88 |
| 07.08.1998 | 0.89 | 0.93 | 0.96 | 0.97 | 0.99 | 1.00 | 0.96 |
| 10.08.1998 | 0.91 | 0.95 | 0.98 | 0.99 | 0.99 | 0.98 | 0.94 |
| 11.08.1998 | 0.94 | 0.98 | 0.99 | 0.98 | 0.96 | 0.95 | 0.91 |
| 12.08.1998 | 0.85 | 0.90 | 0.92 | 0.93 | 0.95 | 0.96 | 1.00 |
| 13.08.1998 | 0.93 | 0.98 | 1.00 | 0.99 | 0.97 | 0.96 | 0.92 |
| 14.08.1998 | 0.90 | 0.95 | 0.98 | 0.98 | 1.00 | 0.99 | 0.95 |
| 15.08.1998 | 0.94 | 0.98 | 0.99 | 0.98 | 0.97 | 0.95 | 0.92 |
| 17.08.1998 | 0.92 | 0.96 | 0.99 | 1.00 | 0.98 | 0.97 | 0.93 |
| 18.08.1998 | 0.93 | 0.97 | 1.00 | 0.99 | 0.98 | 0.96 | 0.92 |
| 19.08.1998 | 0.94 | 0.98 | 0.99 | 0.98 | 0.96 | 0.95 | 0.91 |
| 20.08.1998 | 0.96 | 1.00 | 0.97 | 0.96 | 0.95 | 0.93 | 0.90 |
| 21.08.1998 | 0.97 | 0.99 | 0.96 | 0.95 | 0.94 | 0.93 | 0.89 |

**Appendix K: Fuzzified Road Accident Data**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Date** | **Training Data Set** | **Fuzzy Set** | **Date** | **Training Data Set** | **Fuzzy Set** |
| 1975 | 1460 | A4 | 1990 | 1574 | A5 |
| 1976 | 1536 | A5 | 1991 | 1471 | A4 |
| 1977 | 1597 | A6 | 1992 | 1380 | A2 |
| 1978 | 1644 | A6 | 1993 | 1346 | A2 |
| 1979 | 1572 | A6 | 1994 | 1415 | A3 |
| 1980 | 1616 | A7 | 1995 | 1228 | A1 |
| 1981 | 1564 | A6 | 1996 | 1122 | A1 |
| 1982 | 1464 | A4 | 1997 | 1150 | A1 |
| 1983 | 1479 | A3 | 1998 | 1224 | A1 |
| 1984 | 1369 | A3 | 1999 | 1173 | A1 |
| 1985 | 1308 | A2 | 2000 | 1253 | A1 |
| 1986 | 1456 | A4 | 2001 | 1288 | A2 |
| 1987 | 1390 | A2 | 2002 | 1145 | A1 |
| 1988 | 1432 | A3 | 2003 | 1035 | A1 |
| 1989 | 1488 | A4 | 2004 | 953 | A1 |

## Appendix L: Fuzzified Student Enrolment Data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Date** | **Training Data Set** | **Fuzzy Set** | **Date** | **Training Data Set** | **Fuzzy Set** |
| 1972 | 13563 | A2 | 1983 | 15497 | A3 |
| 1973 | 13867 | A2 | 1984 | 15145 | A3 |
| 1974 | 14696 | A3 | 1985 | 15163 | A3 |
| 1975 | 15460 | A3 | 1986 | 15984 | A4 |
| 1976 | 15311 | A3 | 1987 | 16859 | A5 |
| 1977 | 15603 | A4 | 1988 | 18150 | A6 |
| 1978 | 15861 | A4 | 1989 | 18970 | A7 |
| 1979 | 16807 | A5 | 1990 | 19328 | A7 |
| 1980 | 16919 | A5 | 1991 | 19337 | A7 |
| 1981 | 16388 | A5 | 1992 | 18876 | A7 |
| 1982 | 15433 | A3 |  |  |  |

**Appendix M: Fuzzified TAIFEX Data**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Date** | **Training Data Set** | **Fuzzy Set** | **Date** | **Training Data Set** | **Fuzzy Set** |
| 2 | 6726.5 | A1 | 10 | 6926 | A5 |
| 3 | 6774.55 | A2 | 11 | 6852 | A3 |
| 4 | 6762 | A2 | 12 | 6890 | A4 |
| 5 | 6952.75 | A6 | 13 | 6871 | A3 |
| 6 | 6906 | A5 | 14 | 6840 | A3 |
| 7 | 6842 | A3 | 15 | 6806 | A2 |
| 8 | 7039 | A7 | 16 | 6787 | A2 |
| 9 | 6861 | A3 |  |  |  |

## Appendix N: Fuzzy Set Group and Respective Optimal Weights for Car Road Accident

**Data**

|  |  |  |
| --- | --- | --- |
| **DATA POINTS** | **MAPS** | **OPTIMAL WEIGHT(S)** |
| 1 | #, #→A5 | #,# |
| 2 | A5→ A6 | #,# |
| 3 | A5, A6→ A7 | 0.022442, 0.977558 |
| 4 | A5, A6, A7→ A7 | 0.23847,0.0023311, 0.81188 |
| 5 | A7, A7→ A6 | 0.98302, 0 |
| 6 | A7, A7 , A6→A7 | 0.47619, 0.28692, 0.25285 |
| 7 | A7, A6, A7→ A6 | 0.45287, 0.08873, 0.43831 |
| 8 | A6, A7, A6→ A5 | 0.93558, 0, 0 |
| 9 | A7, A6, A5→ A5 | 0.081525,0.041537, 0.89499 |
| 10 | A6,A5, A5→ A4 | 0.47014, 0.19109, 0.25047 |
| 11 | A5, A5, A4→ A4 | 0.34784, 0.14902, 0.44225 |
| 12 | A5,A5, A4, A4→ A5 | 0.983841, 0, 0, 0.016159 |
| 13 | A5,A5,A4, A4, A5 →A4 | 0.42501,0, 0.5797, 0,0 |
| 14 | A4, A5, A4→ A5 | 0.21641, 0.7953, 0 |
| 15 | A5, A4, A5→ A5 | 0, 0.076319, 0.96584 |
| 16 | A4, A5, A5→ A6 | 0.46637, 0.40469, 0.25499 |
| 17 | A5, A5 ,A6→ A5 | 0.54514, 0.24358, 0.21717 |
| 18 | A5, A6, A5 → A4 | 0.69881, 0, 0.26337 |
| 19 | A6,A5,A4→ A4 | 0 , 0 , 1 |
| 20 | A6,A5, A4,A4→A5 | 0.32258,0.42598,0.13994,0.082344 |
| 21 | A6,A5,A4,A4,A5→A3 | 0.0073059,0, 0.034287, 0.86937 |
| 22 | A5, A3→ A1 | 0.15219, 0.73786 |
| 23 | A3, A1→A2 | 0.72378, 0.23485 |
| 24 | A1, A2→ A3 | 0.044945, 1 |
| 25 | A1, A2, A3→ A2 | 0.33401, 0.68015, 0 |
| 26 | A3, A2→A3 | 0.99895, 0.025133 |
| 27 | A3, A2, A3→ A4 | 0, 0.13372, 0.92348 |
| 28 | A3, A4→A1 | 0.93486, 0 |
| 29 | A4, A1→A1 | 0.34751, 0.50543 |
| 30 | A1,A1→A1 | 0.023115, 0.82616 |

Appendix O: Fuzzy Set Group and Respective Optimal Weights for Enrolment Data

|  |  |  |
| --- | --- | --- |
| **DATA POINTS** | **MAPS** | **OPTIMAL WEIGHT(S)** |
| 1 | #, #→ A1 | #, # |
| 2 | #, A1 → A2 | #, # |
| 3 | A1 , A1 → A2 | 0 , 0.955055 |
| 4 | A1, A1, A1→A3 | 0.85229, 0, 0.20749 |
| 5 | A1, A2→A3 | 0, 0.56155 |
| 6 | A1, A2, A2→A3 | 0, 0.42887, 0.56148 |
| 7 | A1, A2, A2, A2→A4 | 0.9149, 0, 0, 0.18862 |
| 8 | A2 , A3→A4 | 0.9743, 0.0257 |
| 9 | A3, A4→A5 | 0.052746, 0.947254 |
| 10 | A3, A4, A6→A5 | 0, 0, 0.9743 |
| 11 | A6, A6→A5 | 0.21978, 0.74883 |
| 12 | A6, A5→A3 | 0.14587, 0.79113 |
| 13 | A5, A2→A3 | 0, 0.18867 |
| 14 | A5, A2, A2→A3 | 0, 0.37964, 0.59998 |
| 15 | A5, A2, A2, A2→A3 | 0.80891, 0, 0 |
| 16 | A2, A2, A2, A2→A4 | 0, 0.033894, 0 |
| 17 | A2, A4→ A5 | 0.95174, 0.13425 |
| 18 | A2, A4, A6→A6 | 0.090562,0.28332,0.72234 |
| 19 | A6, A7→A7 | 0.090562,0.28332,0.72234 |
| 20 | A6, A7, A7→A7 | 0.02116, 0, 0.97884 |
| 21 | A6, A7, A7, A7→A7 | 1,0, 0.085242, 0.042223 |
| 22 | A7, A7, A7, A7→A7 | 0, 0, 0.41191, 0.58313 |

## Appendix P: Fuzzy Set Group and Respective Optimal Weights for TAIFEX Data

|  |  |  |
| --- | --- | --- |
| **DATA POINTS** | **MAPS** | **OPTIMAL WEIGHT(S)** |
| 1 | #, #→ A1 | #, # |
| 2 | #, A1 → A1 | #, # |
| 3 | A1,A1→A2 | 0.024726, 0.98242 |
| 4 | A1, A3→A2 | 0, 0.99808 |
| 5 | A3,A2→A6 | 0.074917, 0.95315 |
| 6 | A2, A6→A5 | 0.54409, 0.4641 |
| 7 | A6, A5→A3 | 0.28698, 0.70344 |
| 8 | A6, A5, A4→A7 | 0.987482, 0.012518, 0 |
| 9 | A4, A7→A3 | 0.99808, 0 |
| 10 | A7, A4 → A5 | 0.0092343, 0.987482 |
| 11 | A4, A5→A3 | 0.0987651,0 |
| 12 | A6, A4→A4 | 0.004171, 0.995829 |
| 13 | A4,A5→A3 | 0.99066, 0.010751 |
| 14 | A4, A5, A4→A3 | 0.42951, 0.56504, 0 |
| 15 | A4,A4→A2 | 0.99198,0 |
| 16 | A4,A3→A2 | 0.0017454, 0.995829 |

**Appendix Q: Fuzzy Set Group and Respective Optimal Weights for UNIMAID Data**

|  |  |  |
| --- | --- | --- |
| **DATA POINTS** | **MAPS** | **OPTIMAL WEIGHT(S)** |
| 1 | #,# → A1 | #, # |
| 2 | #, A1 → A1 | #, # |
| 3 | A1, A1→ A2 | 0.654322, 0.345678 |
| 4 | A1, A2→ A2 | 0.54268,0.71685 |
| 5 | A1, A2, A2→A2 | 0, 0.654322, 0.457320 |
| 6 | A1, A2, A2, A2 → A2 | 0.0076546,0.654382,0.10951,0.54268, |
| 7 | A2, A2, A2, A2 → A3 | 0.013258,0.45732,0.653578,0.73428 |
| 8 | A2, A3 → A3 | 0.8431, 0.49361 |
| 9 | A2,A3, A3→ A3 | 0.456781 , 0.322459 , 0.98603 |
| 10 | A2, A3, A3, A3 → A3 | 0.875324,0.555671, 0.234500,0.53987 |
| 11 | A2, A3, A3, A3, A3→ A3 | 0,0.12576,0.025119,0.232678,0.45530 |
| 12 | A3, A3, A3, A3, A3 → A4 | 0.288830, 0, 0.31378, 0.55446, 0.984230 |
| 13 | A3, A4 → A4 | 0, 0.4563210 |
| 14 | A4, A4→A5 | 0.28253, 0.779500 |
| 15 | A4, A5, A5 → A5 | 0.037718, 0.675431 |
| 16 | A4, A5, A5→ A5 | 0.074139, 0.026097, 0.984100 |
| 17 | A5, A5,A5→ A6 | 0.86899, 0.18538, 0.23143 |
| 18 | A5,A6→ A7 | 0.6744321, 0.3766734 |

## Appendix R: Fuzzy Set Group and Respective Optimal Weights for Jigawa Monthly Temperature Data

|  |  |  |
| --- | --- | --- |
| **DATA POINTS** | **MAPS** | **OPTIMAL WEIGHT(S)** |
| 1 | #,# →A1 | #, # |
| 2 | #, A1 → A1 | #, # |
| 3 | A3, A3 → A4 | 0.96888, 0.070553 |
| 4 | A3, A3, A4 → A3 | 0.02464, 0.22839, 0.00271 |
| 5 | A3, A4, A3→ A4 | 0.22839, 0.1745, 0.61559 |
| 6 | A4, A3,A4 → A2 | 0.96888, 0.31326, 0.61191 |
| 7 | A4, A2 → A3 | 0.061413, 0.97133 |
| 8 | A2, A3→ A3 | 0.013474, 0.98158 |
| 9 | A2, A3, A3 → A4 | 0.18357, 0.58789, 0.26039 |
| 10 | A2, A3, A3 ,A4 → A4 | 0.230846, 0.042254, 0.070553 |
| 11 | A3, A4, A4 → A3 | 0.09823, 0.00237, 0.10643 |
| 12 | A4, A4, A3 → A3 | 0.09832, 0.36008, 0.61727 |
| 13 | A4, A3, A3 → A3 | 0.224562, 0.004632, 0.0084507 |
| 14 | A3, A3, A3 → A2 | 0.02245, 0.31325, 0.97183 |
| 15 | A3, A2 → A1 | 0.49387, 0.44118 |
| 16 | A2, A1 → A1 | 0.02321, 0.61559 |
| 17 | A1, A1→ A4 | 0.11145, 0.042264 |
| 18 | A1, A4 → A6 | 0.03267, 0.00256 |
| 19 | A4, A6 → A7 | 0.01897, 0.027108 |
| 20 | A6, A7 → A4 | 0.57553, 0.39372 |
| 21 | A6, A7, A4 → A7 | 0.37410, 0.09821, 0.67198 |
| 22 | A6, A7, A4, A7 → A7 | 0.1745, 0.061413, 0.97133, 0.49387 |
| 23 | A4, A7 ,A7 → A7 | 0.29990, 0.004599, 0.46357 |
| 24 | A7, A7, A7 → A4 | 0.082003, 0.88125, 0.92345 |
| 25 | A7, A7, A4 → A7 | 0.046111, 0.17091, 0.84638 |
| 26 | A7, A7, A4, A7 → A5 | 0.059782, 0.033079, 0.15022, 0.04442 |
| 27 | A7, A5 → A4 | 0.9760, 0.44421 |
| 28 | A5, A4 → A4 | 0.00236, 0.72221 |
| 29 | A5, A4, A4 → A5 | 0.99733, 0.00228, 0.92540 |
| 30 | A4, A5 → A6 | 0.23716, 0.77197 |
| 31 | A5, A6 → A3 | 0.65246, 0.30856 |
| 32 | A6, A3→ A5 | 0.55816, 0.46357 |

**Appendix S: Forecasted Results for Road Accident Data using the Developed Model**

|  |  |  |
| --- | --- | --- |
| **Year** | **Actual Value** | **Forecasted Value** |
| 1976 | 1536.00 | 1538.00 |
| 1977 | 1597.00 | 1606.00 |
| 1978 | 1644.00 | 1650.00 |
| 1979 | 1572.00 | 1566.00 |
| 1980 | 1616.00 | 1613.00 |
| 1981 | 1564.00 | 1570.00 |
| 1982 | 1464.00 | 1455.00 |
| 1983 | 1479.00 | 1490.00 |
| 1984 | 1369.00 | 1381.00 |
| 1985 | 1308.00 | 1317.00 |
| 1986 | 1456.00 | 1452.00 |
| 1987 | 1390.00 | 1377.00 |
| 1988 | 1432.00 | 1438.00 |
| 1989 | 1488.00 | 1474.00 |
| 1990 | 1574.00 | 1584.00 |
| 1991 | 1471.00 | 1476.00 |
| 1992 | 1380.00 | 1384.00 |
| 1993 | 1346.00 | 1358.00 |
| 1994 | 1415.00 | 1415.00 |
| 1995 | 1228.00 | 1224.00 |
| 1996 | 1122.00 | 1124.00 |
| 1997 | 1150.00 | 1150.00 |
| 1998 | 1224.00 | 1231.00 |
| 1999 | 1173.00 | 1176.00 |
| 2000 | 1253.00 | 1244.00 |
| 2001 | 1288.00 | 1280.00 |
| 2002 | 1145.00 | 1154.00 |
| 2003 | 1035.00 | 1025.00 |
| 2004 | 953.00 | 956.00 |

## Appendix T: Forecasted Results for Alabama Student Enrolment Data using the Developed

**Model**

|  |  |  |
| --- | --- | --- |
| **Year** | **Actual Enrolment** | **Forecasted Enrolment** |
| 1972 | 13563.00 | 13678.00 |
| 1973 | 13867.00 | 14197.00 |
| 1974 | 14696.00 | 14701.00 |
| 1975 | 15460.00 | 15247.00 |
| 1976 | 15311.00 | 15204.00 |
| 1977 | 15603.00 | 15966.00 |
| 1978 | 15861.00 | 15484.00 |
| 1979 | 16807.00 | 16841.00 |
| 1980 | 16919.00 | 17031.00 |
| 1981 | 16388.00 | 16528.00 |
| 1982 | 15433.00 | 15300.00 |
| 1983 | 15497.00 | 15176.00 |
| 1984 | 15145.00 | 15290.00 |
| 1985 | 15163.00 | 15091.00 |
| 1986 | 15984.00 | 16056.00 |
| 1987 | 16859.00 | 17045.00 |
| 1988 | 18150.00 | 18521.00 |
| 1989 | 18970.00 | 19403.00 |
| 1990 | 19328.00 | 19018.00 |
| 1991 | 19337.00 | 19117.00 |
| 1992 | 18876.00 | 19102.00 |

## Appendix U: Forecasted Results for TAIFEX Data using the Developed Model

|  |  |  |
| --- | --- | --- |
| **Index** | **Actual Values** | **Forecasted Values** |
| 2 | 6726.50 | 6737.00 |
| 3 | 6774.55 | 6775.00 |
| 4 | 6762.00 | 6779.00 |
| 5 | 6952.75 | 6925.00 |
| 6 | 6906.00 | 6874.00 |
| 7 | 6842.00 | 6851.00 |
| 8 | 7039.00 | 7020.00 |
| 9 | 6861.00 | 6884.00 |
| 10 | 6926.00 | 6946.00 |
| 11 | 6852.00 | 6867.00 |
| 12 | 6890.00 | 6888.00 |
| 13 | 6871.00 | 6845.00 |
| 14 | 6840.00 | 6872.00 |
| 15 | 6806.00 | 6775.00 |
| 16 | 6787.00 | 6814.00 |

**Appendix V: Results for Belgium Car Road Accident Data**

|  |  |  |  |
| --- | --- | --- | --- |
| **Results** | | | |
| **Number of Runs** | **Method(s)** | **RMSE** | **MAPE(%)** |
| 1 | CSO-C & PSO | 11.043 | 0.144 |
| 2 | CSO-C & PSO | 10.754 | 0.133 |
| 3 | CSO-C & PSO | 12.073 | 0.152 |
| 4 | CSO-C & PSO | 10.620 | 0.131 |
| 5 | CSO-C & PSO | 8.912 | 0.111 |
| 6 | CSO-C & PSO | 10.753 | 0.132 |
| 7 | CSO-C & PSO | 14.162 | 0.190 |
| 8 | CSO-C & PSO | 19.162 | 0.232 |
| 9 | CSO-C & PSO | 14.311 | 0.173 |
| 10 | CSO-C & PSO | 14.162 | 0.173 |
| 11 | CSO-C & PSO | 13.372 | 0.161 |
| 12 | CSO-C & PSO | 12.071 | 0.152 |
| 13 | CSO-C & PSO | 11.042 | 0.142 |
| 14 | CSO-C & PSO | 10.753 | 0.133 |
| 15 | CSO-C & PSO | 10.622 | 0.133 |
| 16 | CSO-C & PSO | 8.911 | 0.111 |
| 17 | CSO-C & PSO | 8.644 | 0.133 |
| 18 | CSO-C & PSO | 11.040 | 0.140 |
| 19 | CSO-C & PSO | 10.752 | 0.133 |
| 20 | CSO-C & PSO | 12.860 | 0.192 |
| 21 | CSO-C & PSO | 14.683 | 0.210 |
| 22 | CSO-C & PSO | 14.752 | 0.283 |
| 23 | CSO-C & PSO | 5.931 | 0.346 |
| 24 | CSO-C & PSO | 14.081 | 0.171 |
| 25 | CSO-C & PSO | 14.832 | 0.181 |
| 26 | CSO-C & PSO | 14.743 | 0.180 |
| 27 | CSO-C & PSO | 11.722 | 0.332 |
| 28 | CSO-C & PSO | 5.931 | 0.346 |
| 29 | CSO-C & PSO | 10.623 | 0.323 |
| 30 | CSO-C & PSO | 26.712 | 0.322 |

## Appendix W: Results for Alabama University Student Enrolment Data

|  |  |  |  |
| --- | --- | --- | --- |
| **Results** | | | |
| **Number of Runs** | **Method(s)** | **RMSE** | **MAPE (%)** |
| 1 | CSO-C & PSO | 10.932 | 0.344 |
| 2 | CSO-C & PSO | 10.622 | 0.323 |
| 3 | CSO-C & PSO | 26.711 | 0.322 |
| 4 | CSO-C & PSO | 11.044 | 0.140 |
| 5 | CSO-C & PSO | 10.751 | 0.133 |
| 6 | CSO-C & PSO | 10.620 | 0.134 |
| 7 | CSO-C & PSO | 6.662 | 0.033 |
| 8 | CSO-C & PSO | 8.642 | 0.133 |
| 9 | CSO-C & PSO | 11.044 | 0.144 |
| 10 | CSO-C & PSO | 10.751 | 0.133 |
| 11 | CSO-C & PSO | 12.864 | 0.194 |
| 12 | CSO-C & PSO | 14.682 | 0.211 |
| 13 | CSO-C & PSO | 14.753 | 0.283 |
| 14 | CSO-C & PSO | 13.371 | 0.087 |
| 15 | CSO-C & PSO | 12.072 | 0.015 |
| 16 | CSO-C & PSO | 10.623 | 0.013 |
| 17 | CSO-C & PSO | 11.041 | 0.016 |
| 18 | CSO-C & PSO | 11.162 | 0.018 |
| 19 | CSO-C & PSO | 6.662 | 0.033 |
| 20 | CSO-C & PSO | 10.753 | 0.133 |
| 21 | CSO-C & PSO | 10.624 | 0.131 |
| 22 | CSO-C & PSO | 8.913 | 0.112 |
| 23 | CSO-C & PSO | 10.751 | 0.130 |
| 24 | CSO-C & PSO | 14.160 | 0.191 |
| 25 | CSO-C & PSO | 19.164 | 0.234 |
| 26 | CSO-C & PSO | 14.312 | 0.172 |
| 27 | CSO-C & PSO | 14.163 | 0.173 |
| 28 | CSO-C & PSO | 13.371 | 0.162 |
| 29 | CSO-C & PSO | 12.074 | 0.154 |
| 30 | CSO-C & PSO | 11.044 | 0.145 |

**Appendix X: Results for TAIFEX Data**

|  |  |  |  |
| --- | --- | --- | --- |
| **Results** | | | |
| **Number of Runs** | **Method(s)** | **RMSE** | **MAPE (%)** |
| 1 | CSO-C & PSO | 77.091 | 0.343 |
| 2 | CSO-C & PSO | 60.753 | 0.322 |
| 3 | CSO-C & PSO | 80.072 | 0.322 |
| 4 | CSO-C & PSO | 100.162 | 0.142 |
| 5 | CSO-C & PSO | 60.620 | 0.132 |
| 6 | CSO-C & PSO | 100.711 | 0.133 |
| 7 | CSO-C & PSO | 80.073 | 0.111 |
| 8 | CSO-C & PSO | 100.711 | 0.133 |
| 9 | CSO-C & PSO | 50.913 | 0.142 |
| 10 | CSO-C & PSO | 80.711 | 0.130 |
| 11 | CSO-C & PSO | 80.160 | 0.191 |
| 12 | CSO-C & PSO | 60.373 | 0.212 |
| 13 | CSO-C & PSO | 43.368 | 0.566 |
| 14 | CSO-C & PSO | 77.094 | 0.087 |
| 15 | CSO-C & PSO | 80.081 | 0.015 |
| 16 | CSO-C & PSO | 80.833 | 0.013 |
| 17 | CSO-C & PSO | 80.721 | 0.016 |
| 18 | CSO-C & PSO | 77.072 | 0.018 |
| 19 | CSO-C & PSO | 80.072 | 0.033 |
| 20 | CSO-C & PSO | 100.713 | 0.133 |
| 21 | CSO-C & PSO | 50.913 | 0.132 |
| 22 | CSO-C & PSO | 80.714 | 0.111 |
| 23 | CSO-C & PSO | 80.164 | 0.132 |
| 24 | CSO-C & PSO | 60.373 | 0.190 |
| 25 | CSO-C & PSO | 43.368 | 0.566 |
| 26 | CSO-C & PSO | 77.092 | 0.172 |
| 27 | CSO-C & PSO | 80.083 | 0.172 |
| 28 | CSO-C & PSO | 80.812 | 0.160 |
| 29 | CSO-C & PSO | 80.743 | 0.150 |
| 30 | CSO-C & PSO | 102.164 | 0.140 |

**Appendix Y: Results for UNIMAID Student Enrolment Data**

|  |  |  |  |
| --- | --- | --- | --- |
| **Results** | | | |
| **Number of Runs** | **Method(s)** | **RMSE** | **MAPE (%)** |
| 1 | CSO-C & PSO | 5.931 | 0.340 |
| 2 | CSO-C & PSO | 10.623 | 0.320 |
| 3 | CSO-C & PSO | 8.910 | 0.110 |
| 4 | CSO-C & PSO | 8.643 | 0.130 |
| 5 | CSO-C & PSO | 10.622 | 0.130 |
| 6 | CSO-C & PSO | 8.913 | 0.111 |
| 7 | CSO-C & PSO | 8.640 | 0.133 |
| 8 | CSO-C & PSO | 14.312 | 0.171 |
| 9 | CSO-C & PSO | 14.163 | 0.171 |
| 10 | CSO-C & PSO | 13.372 | 0.160 |
| 11 | CSO-C & PSO | 12.072 | 0.153 |
| 12 | CSO-C & PSO | 11.044 | 0.142 |
| 13 | CSO-C & PSO | 8.913 | 0.111 |
| 14 | CSO-C & PSO | 8.644 | 0.133 |
| 15 | CSO-C & PSO | 2.571 | 0.038 |
| 16 | CSO-C & PSO | 5.933 | 0.342 |
| 17 | CSO-C & PSO | 10.622 | 0.322 |
| 18 | CSO-C & PSO | 8.912 | 0.113 |
| 19 | CSO-C & PSO | 8.644 | 0.133 |
| 20 | CSO-C & PSO | 10.622 | 0.133 |
| 21 | CSO-C & PSO | 8.913 | 0.110 |
| 22 | CSO-C & PSO | 8.643 | 0.132 |
| 23 | CSO-C & PSO | 14.312 | 0.171 |
| 24 | CSO-C & PSO | 8.912 | 0.111 |
| 25 | CSO-C & PSO | 8.644 | 0.131 |
| 26 | CSO-C & PSO | 2.571 | 0.038 |
| 27 | CSO-C & PSO | 5.931 | 0.340 |
| 28 | CSO-C & PSO | 10.623 | 0.320 |
| 29 | CSO-C & PSO | 8.910 | 0.112 |
| 30 | CSO-C & PSO | 2.571 | 0.0375 |

**Appendix Z: Results for Jigawa Monthly Temperature Data**

|  |  |  |  |
| --- | --- | --- | --- |
| **Results** | | | |
| **Number of Runs** | **Method(s)** | **RMSE** | **MAPE(%)** |
| 1 | CSO-C & PSO | 5.381 | 0.342 |
| 2 | CSO-C & PSO | 4.752 | 0.232 |
| 3 | CSO-C & PSO | 5.611 | 0.333 |
| 4 | CSO-C & PSO | 5.123 | 0.289 |
| 5 | CSO-C & PSO | 2.578 | 0.231 |
| 6 | CSO-C & PSO | 0.999 | 0.211 |
| 7 | CSO-C & PSO | 3.541 | 0.222 |
| 8 | CSO-C & PSO | 5.381 | 0.342 |
| 9 | CSO-C & PSO | 5.716 | 0.345 |
| 10 | CSO-C & PSO | 1.235 | 0.201 |
| 11 | CSO-C & PSO | 4.638 | 0.231 |
| 12 | CSO-C & PSO | 4.088 | 0.289 |
| 13 | CSO-C & PSO | 4.666 | 0.299 |
| 14 | CSO-C & PSO | 4.311 | 0.268 |
| 15 | CSO-C & PSO | 2.166 | 0.221 |
| 16 | CSO-C & PSO | 0.357 | 0.133 |
| 17 | CSO-C & PSO | 5.611 | 0.356 |
| 18 | CSO-C & PSO | 5.123 | 0.311 |
| 19 | CSO-C & PSO | 2.578 | 0.231 |
| 20 | CSO-C & PSO | 0.999 | 0.211 |
| 21 | CSO-C & PSO | 3.541 | 0.222 |
| 22 | CSO-C & PSO | 5.381 | 0.342 |
| 23 | CSO-C & PSO | 5.716 | 0.345 |
| 24 | CSO-C & PSO | 1.235 | 0.201 |
| 25 | CSO-C & PSO | 4.638 | 0.23 |
| 26 | CSO-C & PSO | 4.088 | 0.289 |
| 27 | CSO-C & PSO | 4.666 | 0.231 |
| 28 | CSO-C & PSO | 4.311 | 0.268 |
| 29 | CSO-C & PSO | 5.381 | 0.342 |
| 30 | CSO-C & PSO | 4.752 | 0.132 |

## Appendix AA: Complete M-File for the FTS Model

*clr clear*

*format bank D = [*

|  |  |
| --- | --- |
| *1975* | *1460* |
| *1976* | *1536* |
| *1977* | *1597* |
| *1978* | *1644* |
| *1979* | *1572* |
| *1980* | *1616* |
| *1981* | *1564* |
| *1982* | *1464* |
| *1983* | *1479* |
| *1984* | *1369* |
| *1985* | *1308* |
| *1986* | *1456* |
| *1987* | *1390* |
| *1988* | *1432* |
| *1989* | *1488* |
| *1990* | *1574* |
| *1991* | *1471* |
| *1992* | *1380* |
| *1993* | *1346* |
| *1994* | *1415* |
| *1995* | *1228* |
| *1996* | *1122* |
| *1997* | *1150* |
| *1998* | *1224* |
| *1999* | *1173* |
| *2000* | *1253* |
| *2001* | *1288* |
| *2002* | *1145* |
| *2003* | *1035* |
| *2004* | *953* |
| *];* |  |

*m = 7; %cluster\_no*

*[center, U] = clustFun(D, m); [umid, idx]= sort(center(:,2)); U = U(idx,:);*

*F = [];*

*for i = 1:size(U,2)*

*f = U(:,i); [value, index]=max(f); F(i,:) = index;*

*end*

*% Step 3.2: Obtain FSG FSG = fuzzySetGroup(F);*

*% Step 3.3 & 3.4: Obtain & Defuzzify the fuzzy forcast R = {};*

*for i=1:size(D,1) R{i} = [];*

*fuzzySet = FSG(i, find(FSG(i,:)>0)); fuzzySet = fuzzySet(end:-1:1);*

*if ~isempty(fuzzySet)*

*mj = umid(fuzzySet)'; ub = ones(size(mj)); % lb = zeros(size(mj));*

*A = D(i,2);*

*nvars = numel(mj);*

*fun = @(r) (sum(r.\*mj) - A)^2;*

*r = particleswarm(fun,nvars,lb,ub); R{i} = r;*

*x\_hat(i,:) = sum(mj .\* r); end*

*end x\_hat=x\_hat(2:end); figure;*

*plot(D(2:end,1),D(2:end,2), 'b--o', D(2:end,1),x\_hat, 'r--\*')*

*title('Time-invariant Fuzzy Time Series') xlabel('Year')*

*ylabel('Accidents') xlim([1976, 2004])*

*% ylim([13000, 20000])*

*legend({'Actual', 'Forcast'})*

*x = D(2:end,2); clusterCenters = umid membershipValues = U'*

*for i=2:numel(F)*

*fuzzySet = FSG(i,find(FSG(i,:)>0)); fuzzySet = fuzzySet(end:-1:1);*

*display(['Data point', blanks(1),num2str(i), blanks(1),...*

*'maps',blanks(1), num2str(fuzzySet), blanks(1),...*

*'to', blanks(1),num2str(F(i)),blanks(1)...*

*'with weights ',blanks(1), num2str(R{i})]);*

*end*

*dataValue = [D(2:end,1) D(2:end,2) x\_hat]*

*RMSE = sqrt(sum((x-x\_hat).^2)/numel(x)) MAPE = mean(((abs(x-x\_hat)./x).\* 100))*